EXTRACTION OF QCD PARAMETER Λ FROM NEUTRINO-NUCLEON DIS EXPERIMENTS

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1 Introduction

1.1 OVERVIEW

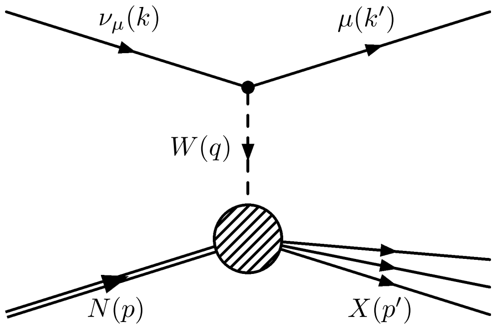
- The extraction is made by fitting the QCD predictions for nucleon structure functions to the structure functions measured from neutrino-nucleon deep-inelastic scattering (DIS).

Organization of the talk is as following:

1. Theory of $\nu - N$ scattering.
2. Brief information about the CCFR experiment.
3. The extraction of Λ.
4. Results, comparisons, conclusions.

1.2 Neutrino-Nucleon Charged-Current Scattering

$$\nu_\mu + N \to \mu^- + X,$$
$$\overline{\nu}_\mu + N \to \mu^+ + X,$$



where $\quad q = k - k' = p' - p$

Invariant kinematic variables used to describe the interaction:

- $Q^2 = -q^2$, the negative square of the four-momentum transfer,

- $x = \frac{Q^2}{2p \cdot q}$, the Björken scaling variable, which represents the fraction of the nucleons's momentum carried by the struck parton, and

- $y = \frac{p \cdot q}{p \cdot k}$, the inelasticity (in the lab frame).

1.3 Differential cross-section of the $\nu-$N scattering

$$\frac{\mathrm{d}^2\sigma^{\nu(\bar{\nu})}}{\mathrm{d}x\,\mathrm{d}y} = \frac{G_F^2 M_N E_\nu}{\pi(1+Q^2/M_W^2)^2}\times$$
$$\left[\frac{1}{2}y^2\cdot 2\mathbf{xF_1}(\mathbf{x},\mathbf{Q^2}) + \left(1-y-\frac{M_N xy}{2E_\nu}\right)\mathbf{F_2}(\mathbf{x},\mathbf{Q^2}) \pm \left(y-\frac{1}{2}y^2\right)\mathbf{xF_3}(\mathbf{x},\mathbf{Q^2})\right]. \quad (1)$$

where $+(-)$ corresponds to neutrino(antineutrino) scattering.

Note that the structure function xF_3 has no counterpart in charged lepton scattering.

1.4 PDFs

Let

$$\mathbf{q} = \begin{pmatrix} u \\ d \\ s \\ \vdots \end{pmatrix}$$

It is convenient to express the quark distributions in flavor combinations. The flavor singlet and nonsinglet distributions are defined as:

$$q^S = \sum_i (q_i + \bar{q}_i),$$
$$q^{NS} = \sum_i (q_i - \bar{q}_i) = \sum_i q_v.$$

where q_v represents the valence quark distribution of proton.

It is customary to define the *isoscalar neutrino structure functions* $2xF_1^N$, F_2^N and xF_3^N, which are the averages of the proton and the neutron structure functions $(F_i^{\nu p} + F_i^{\nu n})/2$:

$$\begin{aligned} 2xF_1^{\nu N} =& F_2^{\nu N} = 2xF_1^{\bar{\nu} N} = F_2^{\bar{\nu} N} \\ &= \sum_i (q_i + \bar{q}_i) \equiv \mathbf{q^S}. \end{aligned}$$

In this study, average of the xF_3 structure function data obtained from neutrino and antineutrino scattering were analyzed. Note that the averaged structure function corresponds to the valence quark distribution:

$$xF_3 = \frac{xF_3^{\nu N} + xF_3^{\bar{\nu} N}}{2} = xu_v + xd_v = x \sum q_v \equiv \mathbf{xq^{NS}}.$$

1.5 The Running Coupling Constant

In the leading logarithmic approximation, the running coupling constant of QCD is written as:

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\ln\left(\frac{Q^2}{\mu^2}\right)}, \quad (2)$$

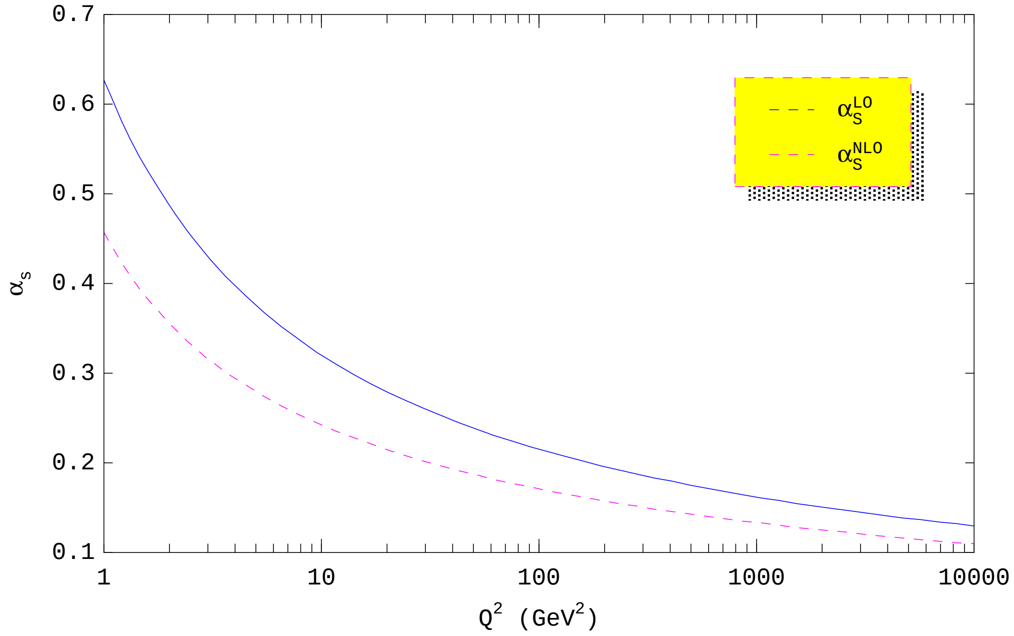
here μ is the renormalization scale and n_f is the number of quark flavors participating in the interaction at the given Q^2 scale.

With the definition of the parameter Λ

$$\Lambda^2 = \mu^2 \exp\left[\frac{-12\pi}{(33 - 2n_f)\alpha_s(\mu^2)}\right],$$

which has the dimension of mass, equation (2) becomes:

$$\boxed{\alpha_s(Q^2) = \frac{12\pi}{(33 - 2\,n_f)\ln(\frac{Q^2}{\Lambda^2})}.}$$



$\alpha_s(Q^2)$ plotted for $\Lambda = 300$ MeV. Dashed line represents the next to leading-order running coupling constant.

1.6 Evolution equations

In perturbative QCD, scaling violations are described by the Altarelli-Parisi (AP) evolution equations:

$$\frac{\mathrm{d}q^{NS}(x,Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y} q^{NS}(y,Q^2)P_{qq}(x/y),$$

$$\frac{\mathrm{d}q^{S}(x,Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y}\left[q^{S}(y,Q^2)P_{qq}(x/y) + G(y,Q^2)P_{qG}(x/y)\right],$$

$$\frac{\mathrm{d}G(x,Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y}\left[q^{S}(y,Q^2)P_{Gq}(x/y) + G(y,Q^2)P_{GG}(x/y)\right].$$

Note that these equations do not predict $p_i(x,Q^2)$ directly but predict how a given $p_i(x,Q_0^2)$ evolves to a different Q^2 scale, where Q_0^2 is some reference scale. Also note that the first equation is independent of the gluon distribution.

2 CCFR Experiment

In this study, the nucleon structure function data measured by the CCFR experiment are used.

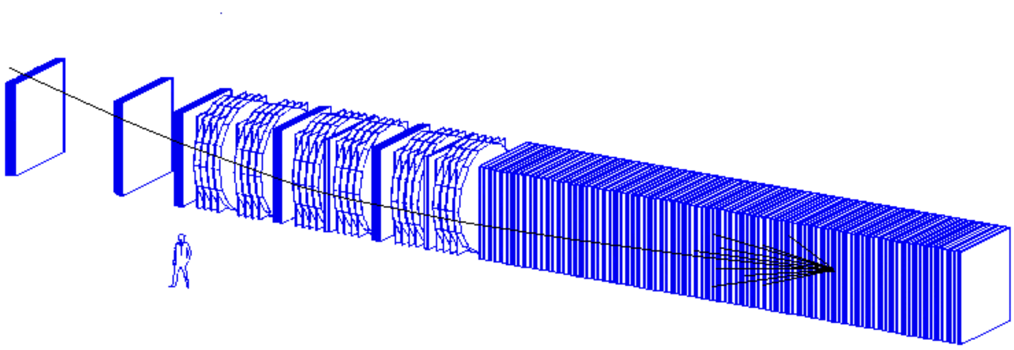
Overview of the experiment

- The CCFR (Chicago-Columbia-Fermilab-Rochester) experiment was performed in the Fermilab (Fermi National Accelerator Facility)

- The data was taken in two experiments, E744 (February-August 1985) and E770 (June 1987-February 1988)

- The detector consists of a (690 ton) iron target-calorimeter and a magnetized toroid spectrometer.

2.1 The Detector



Overview of the CCFR Detector, with a sample neutrino event.

Nucleon structure functions F_2 and xF_3 are measured from neutrino-nucleon scattering by using the number of neutrino and antineutrino events:

$$N^{\nu,\bar{\nu}}(x, Q^2) = \rho L N_A \int_{\text{x-bin}} \mathrm{d}x \int_{Q^2\text{-bin}} \mathrm{d}Q^2 \int_{\text{all energies}} \mathrm{d}E \, \frac{\mathrm{d}\sigma^{\nu,\bar{\nu}}}{\mathrm{d}x \, \mathrm{d}Q^2} \Phi^{\nu,\bar{\nu}}(E),$$

$$\frac{\mathrm{d}\sigma^{\nu,\bar{\nu}}}{\mathrm{d}x \, \mathrm{d}Q^2} = \frac{G^2}{2\pi x} \left[\frac{y^2}{2} 2xF_1 + \left(1 - y - \frac{M_N x y}{2E}\right) F_2 \pm y \left(1 - \frac{y}{2}\right) xF_3 \right],$$

3 Measurements of Λ

3.1 The computer program

- The program is written on a `RedHat Linux 7.3` system, in `Fortran` language.

- The program consists of the QCD model, and the fitting, plotting, reporting machinery.

- The fits are performed with the χ^2 minimization technique, making use of the `CERNLIB`'s `MINUIT` package.

3.1.1 QCD Model

- The QCD model provides the nucleon structure functions F_2 and xF_3 as predicted by QCD.

- The Q^2 dependence of the singlet and nonsinglet distributions are calculated by direct integrations of the AP evolution equations.

- F_i are found from the corresponding evolved singlet and nonsinglet quark distributions, as in the leading-order QCD, the relations

  $$F_2(x, Q^2) = xq^S(x, Q^2), \quad xF_3(x, Q^2) = xq^{NS}(x, Q^2)$$

  hold.

- Calculated structure functions are corrected to account for the target mass effect.

Initial parton distributions and evolution of parton distributions

- The S/NS parton distributions are evolved to the asked Q^2 scale starting from the following initial parameterizations, by integrating the AP evolution equations:

$$
\begin{aligned}
xq^{NS}(x,Q_0^2) &= A_{NS}\, x^{\eta_1}(1-x)^{\eta_2}, \\
xq^{S}(x,Q_0^2) &= A_S(1-x)^{\eta_S} + xq^{NS}(x,Q_0^2), \\
xG(x,Q_0^2) &= A_G(1-x)^{\eta_G},
\end{aligned}
$$

here A_{NS}, η_1, η_2, A_S, η_s, A_G, and η_G are the fit parameters, hence this is the point where the fit parameters other then Λ are introduced into the calculation.

These parameterizations are suggested by the expected behavior of the nonsinglet, singlet and the gluon distributions in the $x \to 0$ and $x \to 1$ limits.

- The reference value of Q^2 is taken to be $Q_0^2 = 5$ GeV^2.

Evolution of the structure functions from the reference scale Q_0^2 to an arbitrary scale Q^2 is performed as following:

1. Integrate the right hand side of the relevant AP equation for the 'current' values of the parton distributions. Note that if $Q^2 = Q_0^2$, the initial parameterizations are used.

2. With the left hand side being a derivative with respect to the Q^2 variable, this integral is the slope of the parton distribution in the Q^2 space. Using this slope, evolve the distribution from Q^2 to $Q^{2'}(= Q^2 + \mathrm{d}Q^2)$ by the *predictor-corrector* method.

3. Repeat these steps until the desired Q^2 value is reached.

4. Program speeds up calculations by creating a matrix of all function values, then returning the asked value by interpolation.

3.2 The fits

The computer program is written to perform two fits

1. The *four parameter* xF_3-only fit.

   Advantage: The nonsinglet distribution does not depend on the gluon distribution $G(x)$, hence is independent of the assumptions and parameterizations made to describe the gluon distribution.

   Disadvantage: xF_3 data has higher statistical errors compared to the F_2 data.

2. The *eight parameter* $F_2 - xF_3$ combined fit.

   Advantage: Introduction of the higher-precision F_2 data into the fit improves the statistical precision of the fit, also providing information about the gluon distribution.

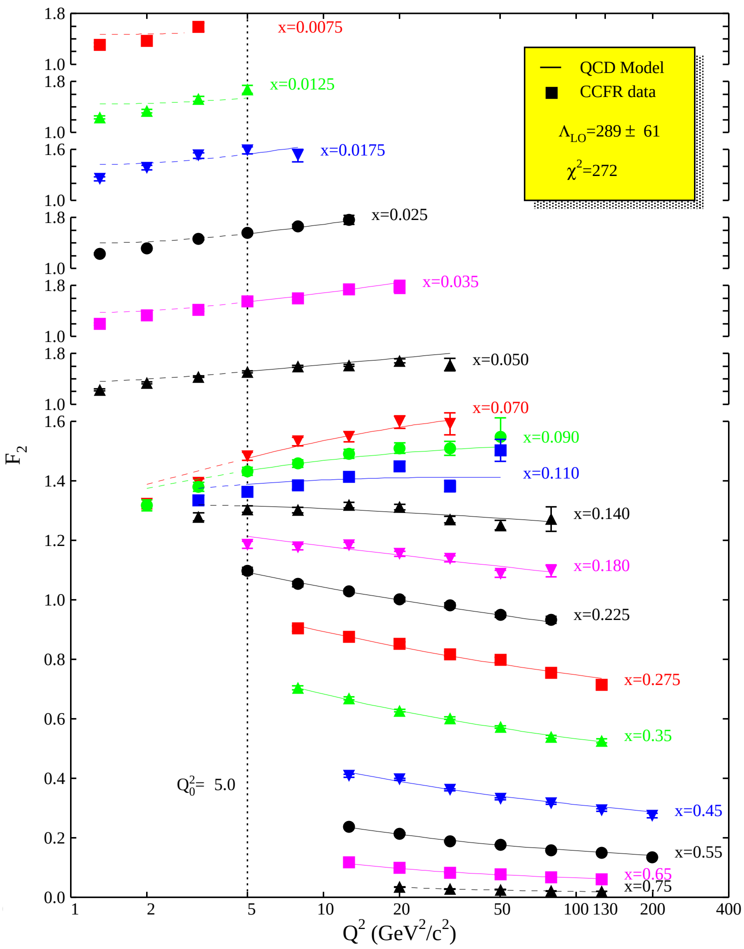
   Disadvantage: Increased number of fit parameters.

3.3 Fit results

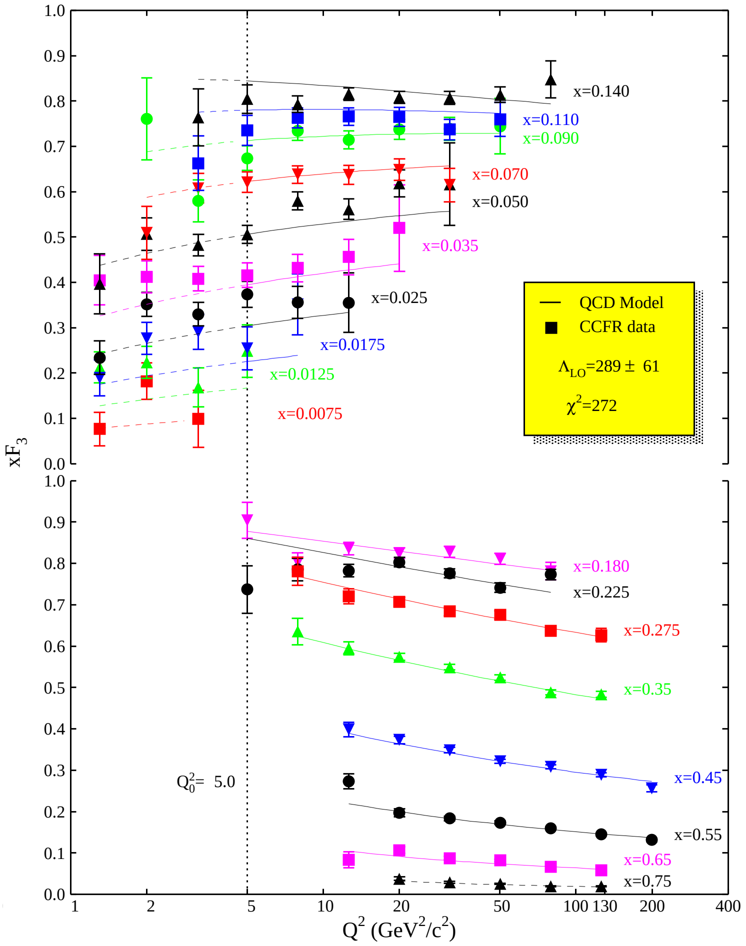
Parameter	xF_3-only fit	Combined fit
Λ (MeV)	$296\,^{+112}_{-99} \pm 112$	$289\,^{+62}_{-59} \pm 76$
A_{NS}	$7.43 \pm 0.49 \pm 0.34$	$7.85 \pm 0.50 \pm 0.32$
η_1	$0.83 \pm 0.03 \pm 0.01$	$0.86 \pm 0.03 \pm 0.01$
η_2	$3.50 \pm 0.11 \pm 0.16$	$3.58 \pm 0.07 \pm 0.13$
A_S		$1.52 \pm 0.06 \pm 0.07$
η_S		$8.05 \pm 0.38 \pm 0.30$
A_G		$4.11 \pm 0.95 \pm 0.98$
η_G		$3.69 \pm 0.89 \pm 0.50$
χ^2/DOF	128/82	272/164
$\alpha_s(M_Z^2)$	$0.131 \pm 0.008 \pm 0.008$	$0.131 \pm 0.004 \pm 0.004$

- The statistical errors of fit parameters are higher in the xF_3-only fit. This is due to the xF_3 structure function data having a higher statistical error compared to F_2.
- The results found from the two fits are consistent.
- QCD predictions for the scaling violations are in agreement with the experimental data (see the plots $\gg$).

Combined fit results for F_2. Dashed lines represent extrapolation of the model.



Combined fit results for xF_3. Dashed lines represent extrapolation of the model.



3.3.1 Logarithmic slopes

- For comparison, the logarithmic slopes $\frac{\mathrm{d}\ln F_i}{\mathrm{d}\ln Q^2}$ of the data and the QCD model are calculated. This provides a well visual summary of the agreement between the data and the model.

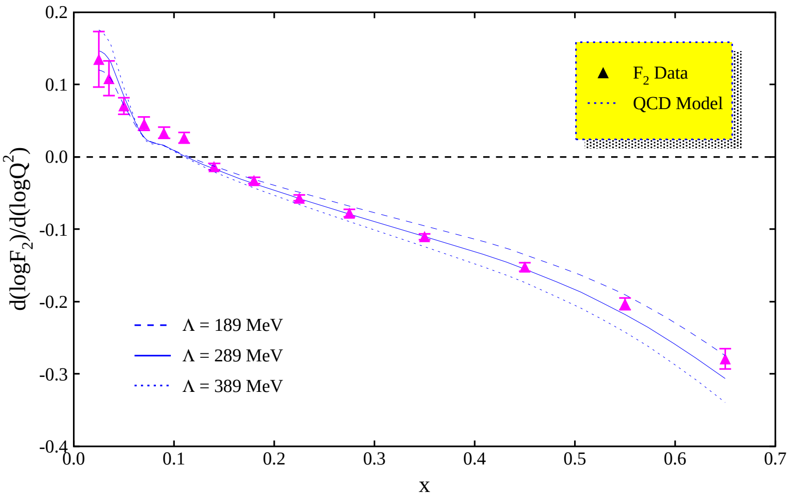
- The logarithmic slopes of the structure function data is found by fitting each x-bin of the structure function data to the following Q^2 parameterization:

$$F_i(Q^2) = A \times (Q^2)^C,$$

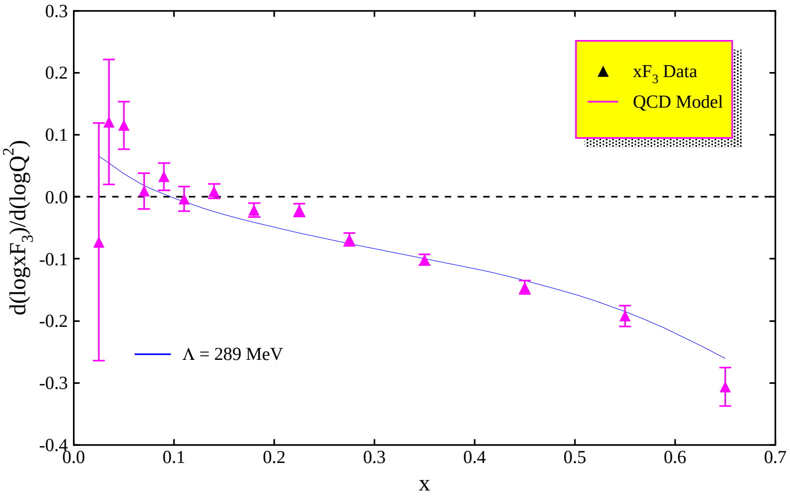
  where A and C are the fit parameters.

- It's easy to notice that the logarithmic slope is nothing but the fit parameter C:

$$\frac{\mathrm{d}\ln F_i(Q^2)}{\mathrm{d}\ln Q^2} = C$$



0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x
d(logF2)/d(logQ2)
F2 Data
QCD Model
Λ = 189 MeV
Λ = 289 MeV
Λ = 389 MeV



0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x
d(logxF3)/d(logQ2)
xF3 Data
QCD Model
Λ = 289 MeV

- Both cases the logarithmic slopes of the structure function data and the QCD model are in good agreement.

- It can be seen that Λ_{LO} is mainly determined by the high-x data.

- Inspecting the AP equations we note that for a particular value of x, the integrands on the right hand side should vanish.

  The plots verify this prediction, the slope of xF_3 vanishes for $x \approx 0.10$ and the slope of F_2 vanishes for $x \approx 0.12$.

Q^2 Cut	Λ_{LO}	χ^2/DOF
$Q^2 > 1$	290 ± 40	537/212 (2.5)
$\mathbf{Q^2 > 5}$	$\mathbf{289 \pm 61}$	$\mathbf{272/164\,(1.7)}$
$Q^2 > 10$	297 ± 69	$170/118\,(1.4)$
$Q^2 > 15$	342 ± 92	122/92 (1.3)

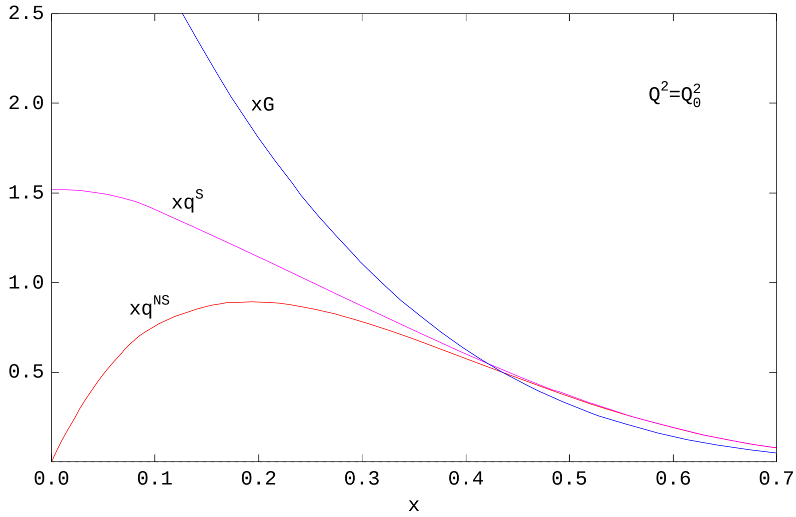
The Q^2 cuts

- From the results of the fits that are repeated for various Q^2 cuts, it's seen that the Q^2 cuts affect the measured Λ_{LO} values.

- The $Q^2 > 1$ cut results in a poor fit, this is due to the inclusion of the data belonging to the non-perturbative region.

  Note that the Λ_{LO} result of this fit is almost the same as the result found for $Q^2 > 5$ cut.

- Also note that higher Q^2 cuts suggest higher values of Λ_{LO} and lower χ^2/DOF ratios, with the statistical precision getting lost.



- All distributions vanish for $x = 1$, as in this limit the observed entity is the target nucleon itself, as a whole.
- The sea quark domination as $x \to 0$ determines the vanishing of the nonsinglet distribution.
- In this limit, the singlet distribution takes a finite value, since this distribution includes contributions from the sea quark distributions.
- The gluon distribution, which is measured as $xG(x, Q_0^2) = (4.11 \pm 1.36)(1-x)^{3.69\pm1.02}$, also converges to a finite value in this limit.

4 Comparisons, Conclusion

4.1 Comparisons

Experiment	Q^2Range (GeV^2)	Λ_{LO} (MeV)	$\Lambda_{\overline{MS}}$ (MeV)
CDHS	1-180	290 ± 30	300 ± 80
CHARM	3-78	$190\ ^{+70}_{-40}$	310 ± 150
CCFR 616/701	1-200	300 ± 100	340 ± 110
BEBC	1.5-55	180 ± 60	–
Average:		233	317
CCFR 744/770	1.3-200	$289\ ^{+62}_{-59}$	381 ± 23

- Measurements of the QCD parameter Λ quoted from various neutrino-DIS experiments, with corresponding statistical errors are tabulated in the table.

- Here $\Lambda_{\overline{MS}}$ refers to a NLO measurement in the minimal subtraction renormalization scheme.

4.2 Conclusion

- This study provided measurements of (i) the QCD parameter Λ, (ii) the strong coupling constant and (iii) the parton distributions of nucleon.

- The Q^2 dependence of structure functions predicted by QCD is found to be consistent with the experimental data.

- The QCD parameter Λ_{LO} is measured by two methods, the *nonsinglet fit* (xF_3 only fit) and the $F_2 - xF_3$ *combined fit*.

- Since the amount of data entering the combined fit is doubled, and the F_2 data have a higher precision than the xF_3 data, the combined fit has a statistical advantage over the xF_3-only fit, yielding results with higher statistical precision. The combined fit also provided information about the gluon distribution.

- The nonsinglet analysis yielded the value

  $\Lambda_{\rm LO}= 296\,^{+112}_{-99}\,\pm 112$ MeV,

  and the singlet analysis yielded

  $\Lambda_{\rm LO}= 289\,^{+62}_{-59}\,\pm 76$ MeV.

  The results were found to be consistent, hence the value with smaller statistical error,

  $$\Lambda_{\rm LO}= 289\,^{+62}_{-59}\,\pm 76\ \textbf{MeV}$$

  is reported as the result of this study.

- This value corresponds to the measurement of the strong coupling constant, $\alpha_s(M_Z^2) = 0.131 \pm 0.004 \pm 0.004$.

- Comparisons show that the $\Lambda_{\rm LO}$ value found in this analysis is comparable to the previous measurements of this parameter.

THE END