A STOCHASTIC MEAN-FIELD APPROACH FOR NUCLEAR DYNAMICS
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--Summer School VI on Nuclear Collective Dynamics--

• Introduction to Nuclear Transport Theory
• Initial Fluctuations: A Stochastic Model for Fusion
• Stochastic Mean-Field Approach
• Dispersion of observables: Lipkin-Meshkov Model
• Transport Coefficients for Relative Momentum and Nucleon Exchange in Deep-Inelastic Heavy-Ion Collisions
• Spinodal Instabilities in Nuclear Matter
• Conclusions
Goal $\Rightarrow$ By truncating many-body evolution, derive an effective transport equation for single-particle density matrix $\rho(\vec{r}, \vec{r}', t)$ or phase-space density $f(\vec{r}, \vec{p}, t)$ (BBGKY Hierarchy)

- Mean-Field approximation (lowest order truncation)
  (Quantal: TDHF or Semi-classical: Vlasov)
many-body wave function $\rightarrow$ single Slater Determinant constructed by single-particle wave functions determined by TDHF equations

\[
i\hbar \frac{\partial}{\partial t} \Phi_{j}(\vec{r}, t) = h(\rho)\Phi_{j}(\vec{r}, t)
\]

\[
\rho(\vec{r}, \vec{r}', t) = \sum \Phi_{j}^{*}(\vec{r}, t) \cdot n_{j} \cdot \Phi_{j}(\vec{r}', t)
\]
\[i\hbar \frac{\partial}{\partial t} \rho(t) = [h(\rho), \rho(t)]\]

At low energies, \(E/A < 10\) MeV, Pauli blocking is very important. With effective interactions mean-field provides good description for average dynamics including one-body dissipation. However collective motions is treated nearly classical approximation! Fluctuations in collective motion are severely underestimated.

Big question: How to incorporate fluctuations into the mean-field description?

- Beyond the Mean-Field:
  Dynamics of density fluctuations play important role in many processes:
  Deep inelastic heavy-ion collisions (radioactive beams)
  Heavy-ion fusion near barrier energies
  Spontaneous and induced fission (symmetry breaking)
  Spinodal instabilities and multi-fragmentations
Heavy-Ion fusion near barrier energies $\rightarrow$ synthesis of super heavy-elements

Induced fission of hot compound nucleus (Stochastic model description of Kramers)
Two different mechanisms for fluctuations:

- **Fluctuations induced by two-body collisions**: collisional mechanism can be incorporated into equation of motion in similar manner to Langevin description of Brownian particle ➔ Semi-classical limit: Stochastic Boltzmann-Langevin Approach [Ayik and Gregoire, PL B212 (1988) 174]

\[
\frac{\partial}{\partial t} f(\vec{r}, \vec{p}, t) - \{ h(f), f(\vec{r}, \vec{p}, t) \} = K(f) + \delta K(t)
\]

- **Mean-Field Fluctuations**: originating from fluctuations in the initial state (quantal and thermal). Dominant mechanism for fluctuations in collective motion at low energies. ➔ Stochastic Mean-Field Approach [Ayik, PL B658 (2008) 174]
INITIAL STATE FLUCTUATIONS: A Classical Stochastic Model For Fusion Near Barrier Energies [Esbensen et al. PRL 41 (1978) 296]

Relative motion is coupled to surface vibrations

\[ H = \frac{\vec{P}^2}{2M} + \frac{\pi^2}{2D} + \frac{1}{2} C\alpha^2 + V(\vec{R}, \alpha) \]

Semi-classical approx for quantal couple-channel calculations
For harmonic modes quantal phase-space distribution \( F(\alpha, \pi, t) \) evolves according to classical Vlasov equation \( \rightarrow \)

\[ \frac{\partial F}{\partial t} - \frac{\partial H}{\partial \pi} \frac{\partial F}{\partial \alpha} + \frac{\partial H}{\partial \alpha} \frac{\partial F}{\partial \pi} = 0 \]

Therefore quantal effects enter through the zero point fluctuations or thermal fluctuation in the initial state
Can incorporate initial fluctuations stochastically by generating an ensemble of classical trajectories.

Surface fluctuations lead to barrier fluctuations, which have a dominant influence on heavy-ion fusion near and sub-barrier energies.
Barrier fluctuations in collisions of nickel ions
Couple-channel calculations with quadrupole and octupole modes [Nobre et al., NPA 786 (2007) 90]

Compared with stochastic classical calculations with same parameters
Ayik, Yilmaz, Lacroix, PRC 81 (2010) 034605
**STOCHASTIC MEAN-FIELD APPROACH**

\[ \Psi(t) = \sum C_{\lambda} \Psi_{\lambda}(t) \quad \Rightarrow \quad \{ \rho_{\lambda}(\vec{r}, \vec{r}', t) \} \]

Generate an ensemble of s.p. density matrices by incorporating density fluctuations (quantal or thermal) in the initial state:

\[ \rho_{\lambda}(\vec{r}, \vec{r}', t) = \sum \Phi_i^*(\vec{r}, t; \lambda) \cdot \rho_{ij}^\lambda \cdot \Phi_j(\vec{r}', t; \lambda) \]

Each matrix element is a Gaussian random number with:

\[ \rho_{ij}^\lambda = \delta_{ij} \cdot n_j \quad \delta \rho_{ij}^\lambda \cdot \delta \rho_{ji}^\lambda = \frac{1}{2} [n_i (1 - n_j) + n_j (1 - n_i)] \]

Single-particle wave functions are determined by the self-consistent mean-field of each event

\[ i\hbar \frac{\partial}{\partial t} \Phi_j(\vec{r}, t; \lambda) = \hbar (\rho_{\lambda}) \cdot \Phi_j(\vec{r}, t; \lambda) \]

One-body dissipation and fluctuations mechanism, consistent with quantal fluctuation-dissipation theorem.
Demonstrations of SMF Approach

• Dispersion of one-body observables: Lipkin-Meshkov Model
  (Connection with Ballian-Veneroni’s variational approach)

• Adiabatic projection on collective variables:
  Generalized Langevin equations for relevant variables
  Random force and dissipation are related in accordance
  with quantal fluctuation-dissipation theorem.
  (Connection with Mori formalism)

• Geometric projection on macroscopic variables
  (Connection with nucleon exchange model)

• Analysis of spinodal dynamics and multifragmentation
  in heavy-ion collisions
LIPKIN-MESHKOV MODEL

N particle are distributed over two N-fold degenerate states separated by an energy ε.

Dispersions of quasi-spin variables →

\[ \Delta_i^2 = \langle \hat{J}_i^2 \rangle - \langle \hat{J}_i \rangle^2 \]

Mean-Field: dispersions remain constant

\[ j_i = \frac{1}{N} \langle \hat{J}_i \rangle \]

\[
\frac{d}{dt} \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \begin{pmatrix} 0 & -1 + \chi j_z & \chi j_y \\ 1 + \chi j_z & 0 & \chi j_x \\ -2\chi j_y & -2\chi j_x & 0 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}
\]

SMF: initial \( j_x \) and \( j_y \) are Gaussian random

\[ \overline{j_x^2} = \overline{j_y^2} = \frac{1}{4N} \]

\[ \chi = V(N - 1)/\varepsilon = 0.5 \]
\[ \hat{J}_z = \frac{1}{2} \sum_{p=1}^{N} \left( c_{+p}^+ c_{+p} - c_{-p}^+ c_{-p} \right) \]
\[ \hat{J}_+ = \frac{1}{2} \sum_{p=1}^{N} c_{+p}^+ c_{-p} \]

HF solution: \(|\Phi> = \prod_{1}^{N} a_{0p}^+ |->\)

\[ a_{0p}^+ = c_{-p}^+ \cos \alpha + c_{+p}^+ e^{i\varphi} \sin \alpha \]

\[ E = -\frac{\varepsilon N}{2} \left\{ \cos 2\alpha + \frac{\chi}{2} \sin^2 2\alpha \cos 2\varphi \right\} \]

INITIAL STATE: HF ground state

\[ j_z = \frac{1}{2N} \sum_{j=1}^{N} \left( \rho_{+j,+j} - \rho_{-j,-j} \right) \]
\[ j_+ = \frac{1}{N} \sum_{j=1}^{N} \rho_{-j,+j} \]

\[ \overline{j_z} = -\frac{1}{2} \quad \overline{\delta j_z \cdot \delta j_z} = 0 \]
\[ \overline{\delta j_+ \cdot \delta j_-} = \frac{1}{2N} \]
\[ j_x = \frac{1}{2} (j_+ + j_-) \]
\[ j_y = \frac{1}{2i} (j_+ - j_-) \]
Dispersions of quasi spin operators: averaged over $10^5$ events

$$
\Delta_i^2 = (\overline{J_i})^2 - (\overline{J_i})^2
$$

\[\chi = 0.5\]

\[\chi = 1.8\]

\[\chi = 5.0\]
ADIABATIC PROJECTION OF SMF ON SLOW COLLECTIVE VARIABLES

MORI FORMALISM: Langevin Dynamics
Evolution of relevant collective variables have a stochastic nature described by “Generalized Langevin Equations”
Coupling between relevant variables and intrinsic degrees of freedom have two different but intimately related effects:
→ Dissipation of energy of collective motion (described by friction or collision terms)
→ Dynamical fluctuations of collective variables (described by random force terms)

Random force and dissipation properties should be related in accordance with Fluctuation-Dissipation Theorem:
No dissipation occur without fluctuations
Heavy-Ion fusion near barrier energies → synthesis of super heavy-elements

Induced fission of hot compound nucleus (Stochastic model description of Kramers)
LANGEVIN DESCRIPTION OF BROWNIAN MOTION

Motion of a collective variable interacting with an environment (dust particle in air, deformation variable in nuclear fission) is determined by a stochastic differential equation:

\[ M \ddot{q} + \frac{dU}{dq} = -\beta \dot{q} + \xi(t) \]

Effects of environment:
- Friction force \( \rightarrow \) dissipation
- Random force \( \rightarrow \) fluctuations

Gaussian white noise with
- zero mean \( \overline{\xi(t)} = 0 \)
- second moment \( \overline{\xi(t)\xi(t')} = 2\beta T \delta(t-t') \)

Induced fission (Kramer)

Generating sufficient number of events, we can determine probability distribution of collective variables.
Consider single slow collective variable and introduce quasi-static s.p. representation

\[ q(t) \rightarrow h(q)\Psi_j(\vec{r}; q) = \varepsilon_j(q)\Psi_j(\vec{r}; q) \]

\[ \rho(\vec{r}, \vec{r}', t; q) = \sum_{kl} \Psi_k^*(\vec{r}; q) \cdot \rho_{kl}(t) \cdot \Psi_l(\vec{r}'; q) \]

Can deduce equation of motion of collective variable from energy conservation \( \rightarrow \)

\[ E = trT \cdot \rho(t; q) + \frac{1}{2} tr\rho(t; q) \cdot V \cdot \rho(t; q) \]

\[ \frac{dE}{dt} = \dot{q} \frac{\partial E}{\partial q} + \sum_k \varepsilon_k(q) \frac{\partial}{\partial t}\rho_k(t) = 0 \]
Master equation for occupation factors

$$\frac{\partial}{\partial t} \rho_k(t) = \int_{t_0}^t dt' \sum K_{kj}(t,t')[\rho_k(1-\rho_j) - \rho_j(1-\rho_k)]_{t'} + \delta K_k(t)$$

$$K_{kj}(t,t') = G_{jk}(t,t')v_{kj}(t)v_{jk}(t') + c.c.$$  

Stochastic part is determined by initial correlations

$$\delta K_k(t) = \sum G_{jk}(t,t_0)v_{kj}(t)\rho_{jk}(t_0) + c.c.$$  

Coupling matrix elements $\rightarrow$  

$$v_{kj}(t) = \frac{\dot{q}(t)}{\varepsilon_{kj}} < k \mid \frac{\partial h}{\partial q} \mid j >$$

Propagator $\rightarrow$  

$$G_{jk}(t,t') = \exp[-\frac{i}{\hbar} \int_{t'}^t \varepsilon_{jk}(s)ds]$$

Collision term and its stochastic part both non-Markovian, involves memory effects
Decay time of memory kernel is determined by correlation time of coupling matrix elements \( \tau_c = \hbar / \Delta \)

Weak coupling limit \( \Rightarrow \tau_c \ll \tau_{relax}, \tau_{coll} \)

For parabolic potential well or barrier (local harmonic) memory effects can be approximately incorporated using identity

\[
E(q) = \mp \frac{1}{2} Kq^2 = \mp \frac{1}{2} M\Omega^2 q^2
\]

\[
\delta(t_1) = \delta(t) \cos \Omega(t-t_1) + \frac{1}{\Omega} \delta(t) \sin \Omega(t-t_1)
\]

\( \Rightarrow \) Quantum Langevin Equation

\[
M\ddot{q} + \frac{1}{2} \frac{dM}{dq} \dot{q}^2 + \frac{dE}{dq} = -\gamma \dot{q} + \xi(t)
\]
QUANTUM LANGEVIN EQUATION

Frequency dependent (Cranking) inertia

\[ M(\Omega) = 2\hbar \sum \left| \langle k \mid \frac{\partial}{\partial q} \mid j \rangle \right|^2 \frac{\varepsilon_{jk}}{(\varepsilon_{jk})^2 + (\hbar \Omega)^2} \rho_k \]

Frequency dependent one-body friction

\[ \gamma(\Omega) = \sum \left| \langle k \mid \frac{\partial h}{\partial q} \mid j \rangle \right|^2 \frac{1}{\Omega (\varepsilon_{jk} - \hbar \Omega)^2 + \eta^2} (\rho_k - \rho_j) \]

Gaussian random stochastic force (Non-Markovian)

\[ \overline{\xi(t)\xi(t')} = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \hbar \omega \cdot \coth \frac{\hbar \omega}{2T} \cdot \gamma(\omega) \]

In SMF approach, one-body dissipation mechanism appears in accordance with Quantum Fluctuation-Dissipation Theorem

High temperature \( \rightarrow \) (classical limit)

\[ \overline{\xi(t)\xi(t')} \approx 2T \cdot \gamma \cdot \delta(t - t') \]

Probability distribution \( P(q,t) \) is produced by generating an ensemble of trajectories
Linear Langevin equation \( \Rightarrow \) \[ M\ddot{q} + Kq = -\gamma \dot{q} + \xi(t) \]

Analytical solution:
Joint probability distribution of collective variable and associated momentum is Gaussian determined by mean values and variances

\[
\begin{align*}
\dot{p}(t) + Kq(t) &= -\beta p(t) \\
M\dot{q}(t) &= \dot{p}(t) \\
M\dot{\sigma}_{qq}(t) &= 2\sigma_{qp}(t)
\end{align*}
\]

\[
\begin{align*}
\dot{\sigma}_{qp}(t) + K\sigma_{qq}(t) &= \frac{1}{M} \left[ \sigma_{pp}(t) - \beta \sigma_{qp}(t) + D_{qp}(t) \right] \\
\dot{\sigma}_{pp}(t) + 2K\sigma_{qp}(t) &= -2\beta \sigma_{pp}(t) + 2D_{pp}(t)
\end{align*}
\]

Formation probability (probability to cross saddle point)

\[
P_f(t) = \int_0^\infty dq \frac{1}{\sqrt{2\pi}\sigma_{qq}(t)} \exp \left\{ -\frac{[q - \bar{q}(t)]^2}{2\sigma_{qq}(t)} \right\} = \frac{1}{2} \text{erfc} \left[ -\frac{\bar{q}(t)}{\sqrt{2\sigma_{qq}(t)}} \right]
\]

Linear Langevin equation has analytical solution

Probability distribution is Gaussian determined by mean-values and variances of collective variables

Heavy-Ion fusion \( \rightarrow \)
Formation probability (Probability to cross saddle)

\[
P_f(t) = \int_{0}^{+\infty} dq P(q; t) = \frac{1}{2} \text{erfc} \left[ -\frac{\bar{q}(t)}{\sqrt{2} \sigma_{qq}(t)} \right]
\]

Washiyama et. al.
Int. Conf. Venezia, Italy (2006)
From TDHF to macroscopic dynamics

Macro variables: mass-charge asymmetry, relative distance-relative momentum, reduced mass of di-nuclear system
Macro variables: mass and charge of projectile-like and target-like fragments, relative distance and relative momentum are defined by integrating phase-space density $f(x, p_x, t)$ over left and right sides of window.

Langevin description: Fluctuating fluxes across the window act as random forces on macro variables

$$\frac{d}{dt} \left( \frac{\delta P}{\delta A_T} \right) = \int \frac{dp_x}{2\pi\hbar} \frac{p_x}{m} \left( \begin{array}{c} p_x \\ 1 \end{array} \right) \delta f(x, p_x, t) |_{x_0} = \left( \begin{array}{c} \xi_P(t) \\ \xi_A(t) \end{array} \right)$$

(Markovian) Gaussian random forces determined by diffusion coefficients

$$\overline{\xi_\alpha(t) \xi_\alpha(t')} = 2\delta(t - t')D_{\alpha\alpha}(R)$$
Correlation function of reduced phase-space density
(Wigner function) in semi-classical limit

\[ \delta f(x, p_x, t) \delta f(x, p'_x, t') \big|_{x_0} = (2\pi\hbar)^{m} \frac{m}{p_x} \delta(t - t') \delta(p_x - p'_x) \Lambda^+(p_x, t) \]

Relative momentum \( \rightarrow \)

\[ D_{PP}(t) = \int \frac{dp_x}{2\pi\hbar} \frac{|p_x|}{m} \frac{p_x^2}{2} \Lambda^+(p_x, t) \]

Nucleon exchange \( \rightarrow \)

\[ D_{AA}(t) = \int \frac{dp_x}{2\pi\hbar} \frac{|p_x|}{m} \frac{1}{2} \Lambda^+(p_x, t) \]

\[ \Lambda^\pm(p_x, t) = \sum_{\sigma\tau} f_P(p_x, t)[1 - \tilde{f}_T(p_x, t)] \pm f_T(p_x, t)[1 - \tilde{f}_P(p_x, t)] \]

Similar to nucleon exchange model, but more refined and microscopic description of transport coefficients
Phase-space distribution (Wigner function):

\[
f(\vec{r}, \vec{p}, t) = \int d^3q \exp[-i\vec{p} \cdot \vec{q}] \cdot \sum_j \Phi_j(\vec{r} - \frac{\vec{q}}{2}, t)\Phi_j^*(\vec{r} + \frac{\vec{q}}{2}, t)
\]

Reduced phase-space distribution:
(integrated over the window)

\[
f(x, p_x, t) = \int dydz \frac{dp_y dp_z}{(2\pi\hbar)^2} f(\vec{r}, \vec{p}, t)
\]

Phase-space distribution averaged over the window

\[
\overline{f}_{P/T}(x_0, p_x, t) = \frac{1}{\Omega(x_0, t)} f_{P/T}(x_0, p_x, t)
\]

Calculated using 3D TDHF Code with Sly4d (Bonch et al.)
Momentum Diffusion Coefficient

\[ D_{PP}(t) = \int \frac{dp_x}{2\pi\hbar} \frac{|p_x| p_x^2}{m} \frac{1}{2} \Lambda^+(p_x,t) \]

**Einstein’s relation**

\[ D_{PP}^{eq}(t) = \gamma(R)T(t) \]

Ayik, Washiyama, Lacroix, PRC 79 (2009) 054606

3D TDHF Code with Sly4d by Bonch et al.
Transport coefficients have similar structure familiar from nucleon exchange model: But not restricted by adiabatic or diabatic conditions, and provide more refined description.

\[ F(t) = \int \frac{dp_x}{2\pi\hbar} \frac{|p_x|}{m} p_x \Lambda^{-}(p_x, t) = -\beta(R)P \]
Nucleon diffusion coefficient $s$ in central collisions of symmetric systems at below barrier energies

$$D_{AA}(t) = \int \frac{dp_x}{2\pi\hbar} \frac{|p_x|}{m} \frac{1}{2} \Lambda^+(p_x, t)$$

Washiyama, Ayik, Lacroix, PRC 80 (2009) 031602R

3D TDHF Code with Sly4d by Bonch et al
Dispersions of fragment mass distributions in central collisions of symmetric systems at below barrier energies

\[ \sigma_A^2(t) \approx \int_0^t dt' 2D_{AA}(t') \]

Empirical relation familiar from nucleon exchange model

\[ \sigma_A^2(t) = N_{exc}(t) \]
• Heated and compressed matter expands at nearly constant entropy, cools down and enters into spinodal region.
• Uniform matter unstable, small density fluctuations grow rapidly leading the system to break-up into clusters.
• Provides a dynamical mechanism for nuclear fragmentation (signature of liquid-gas transformation of nuclear matter)
Early development of density fluctuations: Linear response treatment of SMF around a reference state  \( \delta \rho(t) = \rho(t) - \rho_0 \)

\[
i\hbar \frac{\partial}{\partial t} \delta \rho(t) = [h(\rho_0), \delta \rho(t)] + [\delta h(t), \rho_0]
\]

Nuclear Matter: Collective modes are plane waves
Using method of one-sided Fourier transform, it is possible to carry out nearly analytical quantal treatment of "spectral intensity", "dispersion relation of unstable collective modes" and "equal time density correlation functions"

\[
\delta \rho(\omega) = \int_0^\infty dt e^{i\omega t} \delta \rho(t)
\]

\[
(\hbar \omega - \varepsilon_{21}) < 2 | \delta \rho(t) | 1 > - < 2 | \delta h(t) | 1 > (\rho_2 - \rho_1) = i \hbar < 2 | \delta \rho(0) | 1 >
\]

\[
\delta n_a(\vec{k}, t) = 2 \int \frac{d^3 p}{(2\pi\hbar)^3} < \vec{p} + \frac{\vec{k}}{2} | \delta \rho_a(t) | \vec{p} - \frac{\vec{k}}{2} >
\]
\[ \delta n_n(k, \omega) = \frac{i}{\varepsilon(\vec{k}, \omega)} \left[ \left( 1 + F^{pp} \chi_p \right) A_n(\vec{k}, \omega) - F^{np} \chi_n A_p(\vec{k}, \omega) \right] \]

\[ \delta n_p(k, \omega) = \frac{i}{\varepsilon(\vec{k}, \omega)} \left[ \left( 1 + F^{nn} \chi_n \right) A_p(\vec{k}, \omega) - F^{pn} \chi_p A_n(\vec{k}, \omega) \right] \]

Susceptibility → \( \varepsilon(\vec{k}, \omega) \)

Landau parameters → \( F^{ab} = \left( \partial U_b / \partial n_a \right)_0 \)

Quantal Lindhard functions for neutrons and protons →

\[ \chi_a(\vec{k}, \omega) = 2 \int \frac{d^3 p}{(2\pi \hbar)^3} \frac{\rho_a(\vec{p} + \hbar \vec{k} / 2) - \rho_a(\vec{p} - \hbar \vec{k} / 2)}{\hbar \omega - \vec{p} \cdot \hbar \vec{k} / m} \]

Can be carried out in non-relativistic and relativistic framework

Equal time density correlation function: Time dependence is found by inverse Fourier transform. Calculated using Cauchy-Residue keeping only collective poles of $\varepsilon(\vec{k}, \omega)$

$$\delta n_a(\vec{k}, t) = \int_C \frac{d\omega}{2\pi} e^{-i\omega t} \delta n_a(\vec{k}, \omega) = \delta n^+_a(\vec{k}) e^{+i\Gamma k} + \delta n^-_a(\vec{k}) e^{-i\Gamma k}$$

$$\sigma_{ab}(|\vec{r} - \vec{r}'|, t) = \delta n_a(\vec{r}, t) \delta n_b(\vec{r}', t) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{r} - \vec{r}')} \tilde{\sigma}_{ab}(\vec{k}, t)$$

**Spectral intensity**

$$\tilde{\sigma}_{ab}(\vec{k}, t)(2\pi)^3 \delta(\vec{k} - \vec{k}') = \delta n_a(\vec{k}, t) \delta n_b(-\vec{k}', t)$$
In quantal calculations short wavelengths are suppressed, unstable modes shift towards longer wavelengths and confined to narrower range as compared to those in semi-classical calculations in both symmetric and asymmetric nuclear matter.

In quantal calculations fluctuations take longer time to develop.
Density correlation function

\[ \sigma_{ab}(|\vec{r} - \vec{r}'|, t) = \delta n_a(\vec{r}, t)\delta n_b(\vec{r}', t) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{r} - \vec{r}')} \tilde{\sigma}_{ab}(\vec{k}, t) \]

Correlation length provides a measure for size of primary fragmentation pattern:

- \(3.5 \text{ fm} \to 0.4n\)
- \(3.0 \text{ fm} \to 0.2n\)
Phase Diagrams

For increasing charge asymmetry, spinodal regions shrinks to smaller size.

Unstable region for each mode is further suppressed by quantal effects.
Perturbation asymmetry:
Proton diffusion from low to high density

\[ I_{pt} = \frac{\delta n_n(\vec{r}, t) - \delta n_p(\vec{r}, t)}{\delta n_n(\vec{r}, t) + \delta n_p(\vec{r}, t)} \]

\[ \bar{I}_{pt} \approx \frac{\sigma_{nn} - \sigma_{pp}}{\sigma_{nn} + 2\sigma_{np} + \sigma_{pp}} \]
Finite Systems: Prepare system in a dilute and hot reference state 
(constraint Hartree-Fock or inflating g.s. by scaling)

Dilute Nuclei \[\Psi_j(r) \rightarrow \Psi_j(r/a)\]

Determine unstable collective modes, growth rates by finite temperature RPA (dominant unstable modes are a few low order multipole modes, \(L=2,3,4,\ldots\))

\[
\delta \rho(\vec{r}, \vec{r}', t) = \sum z_L^+(t) \rho_L^+(\vec{r}, \vec{r}') + z_L^-(t) \rho_L^-(\vec{r}, \vec{r}')
\]

\[
\omega_L = \mp i \Gamma_L \quad \text{and} \quad \rho_L(\vec{r}, \vec{r}')
\]

\[
\omega_L \rho_L - [h_L, \rho] - [h, \rho_L] = 0
\]

[Colonna et al., PRL 88(2002) 122701]
Conclusions

Development of stochastic transport models provides powerful microscopic tools for describing complex nuclear dynamics over a wide range of energy including:

- Dissipation and fluctuation mechanisms in DIC
- Dynamical description of phase transformations
- Incoherent and coherent dissipation mechanisms
- Fusion and induced fission dynamics

Lacroix et al., Prog. Part. Nucl. Phys. 52 (2004) 497