Part I: True/False and Multiple-Choice Questions

Choose True or False for the statements below. In either case, write one short but meaningful sentence underneath to explain why. If you don’t, you will not get the point for that statement.

1. Gravitational force does work on a satellite which is in a circular orbit around the Earth.
   - True
   - False

2. Total energy does not change under Lorentz transformations.
   - True
   - False

3. Electric dipole moment of a point charge placed away from the origin is not zero.
   - True
   - False

4. Expectation value of an observable in an energy eigenstate does not change in time.
   - True
   - False
5. The wavelength of light incident from glass to air increases as it is transmitted into the air.
   - True  - False

6. The frequency of light incident from glass to air decreases as it is transmitted into the air.
   - True  - False

7. In order for total internal reflection to take place, light must be incident from a low index medium on to a high index medium at an angle larger than the critical angle.
   - True  - False

8. When a plane wave passes through an aperture with a size close to the wavelength, the transmitted beam shows angular spreading pattern.
   - True  - False

9. An ideal gas in free expansion in a thermally isolated chamber undergoes an increase in its internal energy due to the increased volume available to the molecules.
   - True  - False

10. The entropy increases monotonically as a function of internal energy for all systems in equilibrium.
    - True  - False

11. Since a black body is a perfect absorber, it does not emit any radiation at any temperature.
    - True  - False

12. For systems whose entropy is bounded (such as spin systems), the entropy decreases with increasing energy after a certain point. This is how we get negative temperatures.
    - True  - False
Part II: Classical Problems

13. Air friction is the dominant energy loss mechanism for cars moving at high speeds. As a result, almost all power used by fast cars is spent on overcoming the air friction forces. To find the power $P$ needed, we simplify the problem by modeling the car as a cylindrical object. Using this model, it can be seen that $P$ essentially depends on three quantities.

One of these quantities is the density of air, $\rho_{\text{air}}$, because the air molecules in the car’s path must be pushed away and their inertia is the cause of the frictional forces. The other quantity is the cross-sectional area, $A$, of the car, because this area determines how much air must be pushed away. The length of the car should therefore be unimportant. Finally, $P$ must depend on the speed, $v$, of the car. As a result, power needed must be a simple function of these three quantities, $P = f(A, \rho_{\text{air}}, v)$.

(a) Find this relationship by using dimensional analysis (i.e., find $P = f(A, \rho_{\text{air}}, v)$ by using only the fact that the units come out correctly.)

(b) Suppose that a car increases its speed from 70 km/h to 140 km/h. How much more power should then be necessary to be used?
14. There is an urban legend claiming that the direction of draining water in washbowls are different in each hemisphere and this is associated with the effect of the Coriolis force. Make a rough estimation of the effect to confirm or refute the legend. Take \( \omega_{\text{Earth}} \simeq 7 \times 10^{-5} \text{ rad/s}. \)

15. A particle with mass \( m_0 \) disintegrates into two identical particles, each with mass \( m \). One of the product particles leaves with a speed of \( 3c/5 \) along the \(+x\)-direction. Parent particle was at rest before the disintegration. Write down the momentum 4-vectors of each particle and by using the conservation laws

(a) find the velocity of the other product particle.

(b) Find the \( m_0/m \) ratio.
16. Consider a particle with mass $m$ in a one-dimensional box of length $L$ (suppose box is given as $0 \leq x \leq L$). Let this particle be in the $n$th energy level. We are interested in the force, $F_n$, applied by this particle on the right wall, i.e., the wall at $x = L$. There are two possible ways of computing this force.

(a) We can treat the particle classically, i.e., think of it as a classical particle having energy $E_n$. This particle hits the right wall periodically. At each collision, it imparts an impulse on the wall, and from the impulse per unit time, we can find the average force between the wall and the particle. Compute the following: (i) the speed, $v_n$, of the particle, (ii) the period, $T_n$, between the collisions with the right wall, (iii) momentum change, $\Delta p_n$, of the particle in each collision, and finally (iv) the average force on the particle. This force is also equal to the force on the wall.

(b) An alternative method uses energy concept. Suppose that the right wall has moved towards right by a small distance $\Delta L$. The work done on the wall by the particle, which is the same as the energy transferred from the particle to the wall, is equal to the energy loss of the particle. Using this principle, express the force $F_n$ as a derivative and compute it. Compare with the semiclassical result obtained above.
17. A circular laser beam of 4.0 mW power with 632.8 nm wavelength and 12.00 mm$^2$ cross-sectional area is normally incident on a still water ($n = 1.330$) surface from air.

(a) Calculate the $E$-field amplitude of the reflected beam from the water surface. ($\varepsilon_0 = 8.850 \times 10^{-12}$ F/m)

(b) What is the wavelength of the transmitted beam?

(c) What is the phase velocity of the transmitted beam?

(d) What is the frequency of the transmitted beam?

18. An LED (light-emitting diode) has an emitting surface of 1.0 mm diameter. Light power of 6.0 mW is collected by a lens with 24 mm focal length and 18 mm diameter, placed at a 120 mm distance from the LED.

(a) Determine the position of the LED image.

(b) Determine the diameter of the LED image.

(c) Determine the intensity of the LED image.
19. The bandgap energy of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ material has a dependency on the Al fraction $x$ as follows: $E_g = 1.424 + 1.427x + 0.041x^2$ (eV). A layer of an unknown composition $\text{Al}_x\text{Ga}_{1-x}\text{As}$ sample is illuminated with light of variable wavelength, and it is found that the transmission through the layer decreases rapidly for wavelengths shorter than 726.5 nm.

(a) Determine the Al fraction $x$ in this particular compound.

(b) Determine the energy bandgap of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ at this composition.

20. In a rotating galaxy, let $v(r)$ denote the speed of stars which are at a distance $r$ from the galactic center and $\rho(r)$ denote the mass density at this distance. Observational measurements indicate that $v(r)$ is (almost) independent of $r$ after certain point. Based on this information, what can you say about the $r$ dependence of $\rho(r)$? For simplicity, assume that the galaxy is spherically symmetric.

Note: Observations of this kind, which are first done in the seventies, led to the idea of dark matter.

May 2017

Part I: True/False and Multiple-Choice Questions

Each correct answer is worth +1 point. Each wrong answer is worth -1 point.

21. The de Broglie wavelength of an electron gives an indication of its size.
   - True  - False

22. When a blackbody’s temperature is doubled, its energy output per unit time is also doubled.
   - True  - False
23. For an electron in $\ell = 3$ in a hydrogen atom, there are 6 possible values (with respect to $z$ axis) for the direction of angular momentum.
   [ ] True  [ ] False

24. A magnetic dipole in a uniform magnetic field will feel a net force (when its orientation is appropriate).
   [ ] True  [ ] False

25. Entropy is an extensive quantity.
   [ ] True  [ ] False

26. The escape speed from Earth is around 2 km/s.
   [ ] True  [ ] False

27. The wavefunctions of bosons are completely anti-symmetric under particle exchange.
   [ ] True  [ ] False

28. A dielectric material responds to an externally applied electric field in a way to decrease it inside the material.
   [ ] True  [ ] False

29. In relativity time-order of some events may be switched. For example, if event-1 happens before event-2 in one frame, it is possible that event-1 may happen later than event-2 in some other frame.
   [ ] True  [ ] False

30. When the current that pass over an inductor decreases, the emf generated in the inductor is in the same direction as the current.
   [ ] True  [ ] False

31. The general solution of an $n$th order ordinary differential equation of a single variable depends on $n$ arbitrary parameters.
   [ ] True  [ ] False

32. Although neutrons are charge neutral, it is possible to produce an electric field by making them move.
   [ ] True  [ ] False

**Part II: Classical Problems**

33. Consider a one solar mass Schwarzschild black hole with the Schwarzschild radius $r_s = \frac{2GM_\odot}{c^2}$ and the horizon area $A_s = 4\pi r_s^2$. The Hawking temperature of the black hole is given as $T_H = \frac{\hbar c^3}{8\pi GM_\odot k_B}$. Considering the black hole as a perfect blackbody and using the Stefan-Boltzmann law, estimate the time that will take for the black hole to evaporate, i.e. radiate all of its mass into energy. How does your answer compare with current age of the universe, which is roughly 14 billion years?

*Recall that Stefan-Boltzmann constant is $\sigma = \frac{\pi^2 k_B^4}{60h^3c^2} = 5.67 \times 10^{-8} W \cdot m^{-2} \cdot K^{-4}$ and $M_\odot = 1.98 \times 10^{30} kg$.*

34. When objects like comets, asteroids, or meteors hit the surface of the Earth, large holes in the shape of circles are formed, known as *impact craters*, one of such is shown in the figure. In the history of the Earth, one of the biggest of them, known as the Kara crater, was assumed to be formed around 70 million years ago in the Yugorsky Peninsula, Russia. The diameter of the Kara crater is estimated to be around 120 km at the time of the impact. We are curious about the energy $E$ of the object giving the Kara crater. Using dimensional analysis, show that the energy $E \propto d^4$.

*Hint: You may assume that the diameter $d = d(E, g, \rho)$ where $g$ is the gravitational acceleration and $\rho$ is the density of the soil.*
35. Consider the one-dimensional potentials given below for three different systems. For each of these potentials, put your answers with explanations to the following questions next to each figure.
(i) State the boundary condition for the wavefunction $\psi(x)$ and its derivative $\psi'(x)$,
(ii) State whether the energy is expected to be continuous or quantized, or both.
(iii) Determine whether each system possesses any symmetry leading to separations of the odd and even solutions.

![Potential Diagrams]

36. Consider a particle of mass $m$ in a one-dimensional box

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ \infty & \text{otherwise}, \end{cases}$$

Let us justify the so-called correspondence principle in the given system.

(a) Treat the problem classically first and calculate the probability for the particle to be in the interval, say, $(0, L/4)$.
(b) Now consider the quantum mechanical probability density in the $n$th state. What is the probability for the particle to be in the same interval, $(0, L/4)$.
(c) Take large $n$ limit of the probability you got in part (b). Compare with your result in part (a).

37. Consider that this exam is given by an instructor who happens to be in a spaceship moving away from Earth with normalized relativistic speed $\beta = \frac{v}{c}$. Here $c$ is the speed of light.

The exam starts on the Earth exactly at the time when the spaceship is passing by the Earth. The exam is scheduled to be over in $T_0$ hours in the instructor’s clock. Hence, the instructor sends a light signal back to the class when his clock reads the elapsed time as $T_0$ and the students stop writing when the signal reached them.

(a) Find an expression for the time the students use for the exam (in terms of $\beta$ and $T_0$)?
(b) If $T_0 = 1$ hour and $\beta = 0.96$, calculate the duration that the students use on the Earth.

38. A proton that is restricted to move between $x = 0$ and $x = L$, has an energy of 500 keV in the first excited state.

(a) What is the width $L$?
(b) What will be the wavelength of the photon that is emitted when the proton jumps to the ground state?
(c) How does the probability of finding the proton in the vicinity of $x = L/2$ change when it makes this jump? Explain your answer, by using sketches if necessary.
39. Which of the following wavefunction(s) have a definite momentum? Find the eigenvalue for that one(s); choose one of the others and explain why it does not have a definite momentum. \((A, B, C, k_1, k_2, k_3\) are constants.)

(a) \(A \sin k_1 x\)
(b) \(B(\sin k_2 x + \cos k_2 x)\)
(c) \(C(\sin k_3 x + i \cos k_3 x)\)

Nov 2016

Part I: True/False and Multiple-Choice Questions

Each correct answer is worth +1 point. Each wrong answer is worth -1 point.

40. In Rayleigh scattering the wavelength of the scattered photon changes.
   ○ True  ○ False

41. Fine-structure splitting is due to the interaction of the magnetic moment of the nucleus with the magnetic field of the atom.
   ○ True  ○ False

42. In a non-dispersive medium the group velocity will be equal to the phase velocity.
   ○ True  ○ False

43. The magnetic field of the Earth has the same value in Ankara as it does at the North Pole.
   ○ True  ○ False

44. The size of an object is a good estimate of the uncertainty in its position.
   ○ True  ○ False

45. When a blackbody’s temperature is doubled, its energy output per unit time is also doubled.
   ○ True  ○ False

46. When an electron and a proton are each accelerated from rest through the same potential difference, they will have the same de Broglie wavelength.
   ○ True  ○ False

47. The frequency of the emitted photon is higher in stimulated emission than in spontaneous emission.
   ○ True  ○ False

48. For two dimensional particle in a box, we can have at most 2 electrons in the first energy level above ground state.
   ○ True  ○ False

49. Any two solutions of time dependent Schrödinger’s equation can be added together to give another possible solution to this equation.
   ○ True  ○ False

50. Any two solutions of time independent Schrödinger’s equation can be added together to give another possible solution to this equation.
   ○ True  ○ False

Part II: Classical Problems
51. (a) Assume that your car of mass $m$ and velocity $v$ collides
   (i) to a hard wall.
   (ii) to another car of mass $2m$ with a velocity $-\frac{v}{2}$.
   (iii) to another car of mass $m/2$ with a velocity $-2v$.

   After the collision explain whether your car keeps moving or not? Explain clearly in which case your car
   would get the most damage. Then decide which option(s) appears to be safer under the circumstances.

(b) A car and a large truck collide head on and stick together.
   (i) Explain clearly what happens in regard to the change of momentum.
   (ii) Explain clearly which one experiences larger acceleration during the collision.

(c) Assume that a incident object hits a stationary one. Would it be possible for the stationary one to
have a larger momentum than the incident one? If not explain why. If so, give a clear example.

52. (a) Utku is on one end of a train of length $L$, moving with respect to a platform, on which Toprak is
at rest. Utku observes two events, one taking place at the middle of the train at time $t_1 = L/c$
and another taking place at the far end of the train at time $t_2 = 3L/4c$. Toprak observes these two
events simultaneously.
   (i) What is the speed of the train with respect to the platform?
   (ii) Is Utku at the front or the rear end of the train? (The train is moving in the forward direction.)
      Explain your answer, no credit will be given without a correct explanation.
      Note: This part can be answered independent of the previous part, but you can use your result
      from previous part if you want.

(b) A system of 4 identical particles are sharing 5 units of energy. What is the probability of a particle
having 2 units of energy, if
   (i) the particles are distinguishable.
   (ii) the particles are indistinguishable with spin 1/2.
   (iii) the particles are indistinguishable with spin 1.

53. The $C/T$ as a function of $T^2$ for Cu is shown in the figure.
   (a) What is the functional form of molar heat capacity for Cu? Write $C$ as a power series in $T$, leaving
the coefficients undetermined.
   (b) What do the terms in this series correspond to? What are the contributing factors?
(c) From the values in the graph, determine the values of these coefficients.

(d) What is the Debye temperature for Cu?

Hint: According to Debye, the lattice contribution to heat capacity at low temperatures is given by

\[ C = \frac{12\pi^4}{5}R \left( \frac{T}{T_D} \right)^3. \]

54. Consider a particle in an infinite potential well

\[ V(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ \infty & \text{otherwise}, \end{cases} \]

with energy levels \( E_n \); and another one in finite potential well

\[ V(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ U_0 & \text{otherwise}, \end{cases} \]

with energy levels \( E'_n \), where \( U_0 > 0 \).

(a) Sketch the first excited state wave functions for these particles.

(b) Using these sketches, argue which particle would have the higher energy.

(c) Using this result, argue which particle would have a higher uncertainty in momentum. Discuss this result’s compatibility with the uncertainty principle.

55. When an object is heated up it can be assumed to emit radiation as a perfect blackbody if its emissivity is equivalent to 1. Using the given graph answer the following questions:

(a) In the Rayleigh-Jeans formula the energy density of cavity radiation is given by

\[ u(\nu) = \frac{8\pi n^3 \nu^2}{c^3} kT, \]

where \( \nu \) is the frequency, \( n \) is the refractive index and \( k \) is the Boltzmann constant. In which part of the spectrum shown in the graph does this formula fail? Explain clearly.

(b) Explain how Planck overcame this dilemma using one or two sentences.

(c) Assuming our Sun is yellow, which curve and corresponding temperature in the above graph is closest to its actual temperature?

56. Consider a linear system comprising one particle with mass \( M \) and two particles with mass \( m \) on either side as shown below (a model for the carbon dioxide molecule). The particles are connected with identical springs with spring constant \( K \) (Do not use energy formalism to solve the following problem).

(a) Considering only the motion of the masses along the line connecting them, write down the equation of motion for each particle.

(b) Find the eigenvalues (normal frequencies): Show that \( \omega = 0 \) is a normal mode of this system. Find the two remaining non-zero normal mode frequencies.

(c) What are the ratios of the amplitudes of the three masses? What kind of motion does the \( \omega = 0 \) solution correspond to?
A CW diode laser has a power output of $P_0$ at a center wavelength of $\lambda_0$. If the beam is circular with a diameter of $d_0$, using the constants $c, \varepsilon_0, \mu_0$ for speed of light, permittivity of free space and magnetic permeability and vector quantities $E_0, B_0$ for the Electric Field and Magnetic Field Strength respectively,

(a) Formulate the intensity of the laser beam if traveling through free space

(b) Formulate the intensity if the beam is traveling inside a slab of glass with index of refraction $n$.

(c) If the beam has a Gaussian $\text{TEM}_{00}$ profile, formulate the intensity of the beam at a distance equivalent to the beam waist ($\omega_0$) from the optical axis?

(d) If the bandwidth of this laser is $\delta \nu$ (in units of Hz). Formulate the coherence length of this laser source.

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**May 2016**

**Part I: True/False and Multiple-Choice Questions**

Each correct answer is worth +1 point. Each wrong answer is worth -1 point.

1. Magnetic force between two moving point charges are always Newton’s third law pairs.
   - True  
   - False

2. If point A is at a lower electrical potential then point B, an electron between them will move towards the point B.
   - True  
   - False

3. Electric flux due to point charge through a sphere of radius $2R$ is twice as great as that through a sphere of radius $R$.
   - True  
   - False

4. The wavefunction of the hydrogen atom is spherical symmetric.
   - True  
   - False

5. In the photoelectric effect, the brighter the light, the greater the current produced.
   - True  
   - False

6. An electric dipole rotating under the torque of an electric force has two equilibrium configurations.
   - True  
   - False

7. Water wave is classified as both a longitudinal and transverse wave while waves on strings are classified as only transverse waves.
   - True  
   - False

8. To perform a Michelson interferometer experiment where the interference between two beams from the same laser is used to observe fringes on a screen the laser used can be unpolarized.
   - True  
   - False
66. The atmospheric hurricanes in the Northern Hemisphere turn counterclockwise due to the centrifugal force.
- True
- False

67. The airplane lift is indeed caused by Newton’s third law.
- True
- False

68. Thermal expansion of solids occur because of asymmetric potential wells where the atoms are confined.
- True
- False

69. The induced electric and magnetic fields are both conservative.
- True
- False

Part II: Classical Problems

70. The expansion of an atomic bomb can be studied by the use of dimensional analysis. Atomic bomb is based on nuclear fission, creating immense burst of energy released to the environment. To study its propagation, let us assume that the atomic bomb has a size \( \ell \) and energy \( E \). Once the explosion takes place, we assume that the effect propagates as a spherical wave of radius, say, \( R \), after the time \( t \) passed. The surrounding air is assumed to have initial pressure \( p \) and density \( \rho \). To simplify the discussion further, assume a point like source since \( \ell \ll R \) as well as \( p \) being ignorable as compared to the huge energy released. Under these assumptions one can write that 
\[
R = F(E, \rho, t).
\]

(a) By using dimensional analysis only, determine the function \( F \).

(b) From time-lapse pictures of the atomic bomb, one may estimate the radius of the wave. Using the data given in the picture together with part (a), estimate the energy released in units of TNT. Note that \( 1 \text{ Joule} = 2.5 \times 10^{-4} \text{g} \times \text{TNT} \). Take \( \rho = 1.25 \times 10^{-3} \text{g/cm}^3 \) for the air.

71. In September 14, 2015, LIGO observatory has detected the gravitational waves for the first time. Although the waves could be detected only for a fraction of a second, it contained enough information to infer that the waves are emitted by two massive bodies, most probably two black holes, each with masses around \( m \sim 30M_\odot \). The frequency of the signal indicates that the bodies rotated around each other with a frequency of \( f_{\text{rot}} \sim 75 \text{ Hz} \). Although the event is highly relativistic and general relativity has to be used to correctly analyze it, we can still get an idea of the immense magnitude of the event by doing a calculation with non-relativistic Newtonian gravity. (As this is only a rough calculation, only an order-of-magnitude calculation is enough.)

(a) Using the given values of frequency and masses, find the radius \( R \) of the orbits of the two bodies (assume that both have equal masses).

(b) Compute the orbital speed \( v \) of the bodies. Are they relativistic?

(c) The radius of the event horizon of black holes is given by the Schwarzschild formula,
\[
R_{\text{Sch}} = \frac{2Gm}{c^2}.
\]
Compute the Schwarzschild radius of the objects and compare with orbital radius \( R \) you have found above.
72. A wire loop of radius \( a \), mass per unit length \( \rho \) is carrying a steady current \( I \). This loop is levitated against its weight at a height \( h \) above the north pole of a very long cylindrical bar magnet with radius \( r \). Assuming that \( r \ll a \) and \( r \ll h \), we can consider the magnetic field of the cylindrical bar magnet is radially outward from its north pole and it may be written as \( B = \frac{1}{4} \mu_0 M \hat{r} \). Determine the magnetization of the permanent magnet in terms of the given quantities and relevant physical constants.

73. In the interstellar space, very high energetic gamma ray photons of energy \( E_{\text{in}} \) interacts with photons of energy \( E \) to produce electron-positron pairs. Take \( m \) as the electron mass.

\[ \text{Hint: The minimum energy condition can be thought of creating electron and positron with the same energy and momenta, both of which are being parallel to the incident photon momentum.} \]

(a) Find the minimum required energy \( E_{\text{in}} \) of the gamma rays to produce a pair of electron-positron.

(b) If \( E = 2.5 \times 10^{-14} \text{ eV} \) (\( \sim 3 \text{ Kelvin} \)), calculate the minimum gamma rays energy \( E_{\text{in}} \) in units of eV. Use 1 Joule \( \simeq 6.2 \times 10^{18} \text{ eV} \).

74. Consider a potential well where the potential \( V \) increases linearly in \( x \) in between \( x = 0 \) and \( x = L \) and it is infinite at the boundaries, as given in the figure. Plot roughly a possible wavefunction as a function of \( x \). Explain with reasoning how the wavelength and the amplitude change.

75. The potential energy of a diatomic molecule where each atom is held by an ionic bond is given by the following formula:

\[ U(r) = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{B}{r^6}. \]

An example of such a molecule is \( \text{HCl} \) (\( \text{H}^+ - \text{Cl}^- \); one with charge \( +e \) and the other with charge \( -e \)). Here, \( r \) is their separation. The first term is due to Coulomb attraction, while the second term is meant to represent the repulsive force between the two ions (\( B \) is a constant). This is a simple approach to approximate in reality this complex term, which states that the repulsive force becomes very important when \( r \) is small.

(a) Calculate \( B \), for stable equilibrium \( (r = r_{\text{eq}}) \).

(b) Prove that this is a stable equilibrium.

(c) Using your approach from parts (a) and (b), and assuming that \( m_{\text{H}^+} \ll m_{\text{Cl}^-} \), calculate the angular frequency of vibration for this potential.

76. For light waves traveling in glass which is an isotropic, homogeneous optical medium with no free charges and no magnetization (glass has the following dielectric properties: \( \epsilon(\omega) = \epsilon_r(\omega)\epsilon_0 \) and \( \mu = \mu_r\mu_0 \) with \( \mu_r = 1 \)):

(a) Write down the wave equation for the \( E \) and \( H \) fields separately in this medium, define all parameters.

(b) What is the phase velocity of the wave; express your answer in terms of the dielectric properties of the medium?

(c) What is the group velocity as a function of the wavelength dependent refractive index \( n(\lambda) \)?

\[ \text{Hint: First start by stating } \omega = k v_p, \text{ and then formulate the first order derivative of } n \text{ with respect to } \lambda. \]

(d) Using your solution in part (c) discuss the behavior of the group velocity for both anomalous and normal dispersive regimes.
77. When a plane wave light source with wavelength, \( \lambda \), is used to illuminate a circular aperture of diameter \( D \), the half beam angular spread of the beam at a screen a distance \( L \) away is given by:

\[
\theta_{1/2} \sim 1.22 \frac{\lambda}{D} .
\]

(a) In the equation above explain in one sentence where the factor of 1.22 comes from?
(b) If a plane wave is incident on a thin lens with focal length, \( f > 0 \), where will the light focus to and what will be the size of the focused spot at that position?
(c) The half beam divergence angle of a fundamental Gaussian laser beam is given by

\[
\theta_{1/2} \sim \frac{\lambda}{\pi \omega_0} ,
\]

where the beam waist radius of the laser beam is given by \( \omega_0 \). If the incident laser beam has its waist at the position of the thin lens with focal length \( f > 0 \), then where does the laser beam focus to and what will be the size of the focused spot at that position?

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**November 2015**

**Part I: True/False and Multiple-Choice Questions**

Each correct answer is worth +1 point. Each wrong answer is worth -1 point.

1. 78. For two particles having spin 1/2, the total spin can be zero.
   - True  
   - False

1. 79. Phase speed of waves can exceed the speed of light.
   - True  
   - False

1. 80. For black-body radiation, the wavelength corresponding to maximum intensity is proportional to the temperature.
   - True  
   - False

1. 81. The third law of thermodynamics states that “negative absolute temperatures” can really exist.
   - True  
   - False

1. 82. For a particle in a box (say, in the ground state), if the box size is reduced to its half, the kinetic energy of the particle increases 4 times.
   - True  
   - False

1. 83. If a particle is confined, its energy becomes quantized.
   - True  
   - False

1. 84. All Bessel functions \( J_\nu(x) \) (\( \nu \) real) have infinitely many zeros.
   - True  
   - False

1. 85. Water wave is classified as both a longitudinal and transverse wave while waves on strings are classified as only transverse waves.
   - True  
   - False

1. 86. The \( s, p, d \) and \( f \) orbital designations for the electron states in an atom correspond to the principal quantum number, \( n = 1, 2, 3, 4 \) levels respectively.
   - True  
   - False
87. For isoelectronic atoms (atoms with only one electron) the energy required to ionize the atom increases with increasing atomic number.
   ○ True   ○ False

88. To perform a Michelson interferometer experiment where the interference between two beams from the same laser is used to observe fringes on a screen the laser used can be unpolarized.
   ○ True   ○ False

89. A change in parity is required for an electron to undergo a radiative transitions between energy levels in an atom.
   ○ True   ○ False

90. In the 17th century, Otto von Grücke, a physicist from Magdeburg, fitted two hollow bronze hemispheres together and removed the air from the resulting sphere with a pump. Two eight-horse teams could not pull the halves apart even though the hemispheres fell apart when air was readmitted. Suppose von Grücke had tied both teams of horses to one side and bolted the other side to a heavy tree trunk. In this case, the tension on the hemispheres would be
   1. twice
   2. exactly the same as
   3. half
   what it was before.

91. You are a passenger in a car and not wearing your seat belt. Without increasing or decreasing its speed, the car makes a sharp left turn, and you find yourself colliding with the right-hand door. Which is the correct analysis of the situation?
   1. Before and after the collision, there is a rightward force pushing you into the door.
   2. Starting at the time of collision, the door exerts a leftward force on you.
   3. Both of the above
   4. Neither of the above.

92. Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Just before hitting the ground, the heavier marble has
   1. As much kinetic energy as the lighter one
   2. twice as much kinetic energy as the lighter one
   3. half as much kinetic energy as the lighter one
   4. four times as much kinetic energy as the lighter one
   5. impossible to determine

93. A stone is launched upward into the air. In addition to the force of gravity, the stone is subject to a frictional force due to air resistance. The time the stone takes to reach the top of its flight path is
   1. larger than
   2. equal to
   3. smaller than

   the time it takes to return from the top to its original position.

94. Suppose Earth had not atmosphere and a ball were fired from the top of Mt. Everest in a direction tangent to the ground. If the initial speed were high enough to cause the ball to travel in a circular trajectory around Earth, the ball’s acceleration would
1. be much less than g (because the ball doesn’t fall to the ground).
2. be approximately g
3. depend on the ball’s speed

95. Think fast! You’ve just driven around a curve in a narrow, one-way street at 50 km/h when you notice a car identical to yours coming straight toward you at 50 km/h. You have only two options: hitting the other car head on or swerving into a massive concrete wall, also head on. In the split second before the impact, you decide to
1. hit the other car
2. hit the wall
3. hit either one—it makes no difference
4. consult your lecture notes.

Part II: Classical Problems

96. Consider a particle of mass $m$ that is released from height $h$ with zero initial speed. Let $T$ be the time it takes to hit the ground and $g$ be the gravitational acceleration. Using dimensional analysis only, show that $T$ is proportional to $\sqrt{h/g}$ and is independent of the mass.

97. Consider a square table of area $\ell^2$ covered with a tablecloth and there is a plate precisely in center of the table. Somebody is trying to pull the tablecloth at a steady rate out from beneath the plate. There is a sliding friction between the plate and the tablecloth (with coefficient $\mu_1$) and between the plate and the table (with coefficient $\mu_2$). As the tablecloth is being pulled, the plate in contact with the cloth moves with a distance $d_1$ and then the distance it continues to move while in contact with the table is $d_2$ before it stops. The gravitational acceleration is $g$.

(a) Express the maximum velocity, $v_{max}$, that the plate can reach in terms of $\mu_1, g$ and $d_1$ by considering the part of motion of the plate together with the tablecloth. Draw the free-body diagram.

(b) Express $v_{max}$ now in terms of $\mu_2, g$ and $d_2$ using the second part of the motion. Draw the free-body diagram.

(c) In order for the plate just to remain on the table, show that the maximal distance it travels with the tablecloth is

$$d_{1max}^\ell = \frac{\mu_2}{2 \mu_1 + \mu_2} \ell.$$

Find also the time it takes to get $d_{1max}^\ell$.

98. Consider a particle in a box with hard-wall boundaries in a potential $V(x)$ as shown below. The energies of the ground state, $E_0$, and the 5th excited state, $E_5$, are shown on this diagram. Carefully sketch plots of the corresponding wavefunction $\psi_0(x)$ and $\psi_5(x)$. 
99. This year’s physics Nobel prize is given for the experimental confirmation of neutrino oscillations. The discovery also implies that the neutrinos have mass. Although the experimental data do not give us the neutrino masses directly, it is possible to infer that the masses could be as low as
\[ m \sim 0.05 \text{eV}/c^2 \sim 10^{-37} \text{kg}. \]
Since they have mass, it is in principle possible to slow down the neutrinos. Perhaps, in one day, it might be possible to capture them inside a small box. In this problem, you are going to estimate the minimum speeds of neutrinos if they were ever captured in such a box.

(a) Suppose the box has a size of \( \ell \sim 1 \text{ m} \). Use the uncertainty principle to find a typical value for the minimum possible momentum \( p \).

(b) Particles are relativistic if \( p \) comparable or bigger than \( mc \). Are these neutrinos relativistic? (Use the mass value given above.)

(c) What will be the minimum possible speed of neutrinos captured in such a box?

100. Consider a particle of mass \( 3m \) at rest. It disintegrates into two daughter particles of the same mass, which might be moving at relativistic speeds. None of the particles has any internal structure.

(a) If the daughter particles have mass \( m \), find their speeds and draw a diagram showing in which way they move.

(b) What would you say about the speeds of the daughter particles if their masses were equal to \( 3m/2 \)?

(c) Is it possible for the daughter particles to have a mass greater than \( 3m/2 \)? Explain your answer.
101. While digital camera form images with pixels, the human eye forms images with rods and cones that lie on the retina. If two point objects close together are to be seen as two distinct objects, the images must fall on the retina on two different cones that are not adjacent. That is, there must be a cone between them. Assume that the cones are sensitive to a wavelength of 550nm. If the iris is about 5mm in diameter, and we can model the human eye as a sphere with roughly a 25mm diameter,

(a) what is the minimum separation between two objects at a distance of 2m away from the eye that we can resolve at this wavelength?

(b) what is, to a linear approximation, the distance between the two cones on the retina?

102. An electromagnetic plane wave from a linearly polarized laser whose electric field wavefunction is given by the following equation:

\[ E(x, t) = E(y, z) \left[ \sin \left( 3 \times 10^6 \pi x + 9 \times 10^{14} \pi t \right) \hat{j} + \sin \left( 3 \times 10^6 \pi x + 9 \times 10^{14} \pi t \right) \hat{k} \right] \]

is sent into the following ideal optical elements. Let the intensity of the incident beam be \( I_0 \). Describe the output intensity and polarization, and give a form for the output electric field wavefunction at the output of these ideal optical elements:

(a) Linear polarizer with transmission axis in the \( \hat{k} \) direction

(b) Quarter waveplate with fast axis in the \( \hat{k} \) direction

(c) Half-wave plate with fast axis in the \( \hat{k} \) direction