MIDDLE EAST TECHNICAL UNIVERSITY

Department of Physics

PhD Qualifying Exam
Miscellaneous Questions

Table of Contents

Table of Contents

1.1 Misc-Nov-2018

Part I: True/False and Multiple-Choice Questions

Choose True or False for the statements below. In **either case**, write **one** short but **meaningful** sentence underneath to explain why. If you don't, you will not get the point for that statement.

[1] 1. In blackbody radiation, when temperature is increased, the power emitted at longer wavelengths *decreases* because the dominant power is now emitted in shorter wavelengths.
◯ True ◯ False

[1] 2. Work done by a kinetic friction force on an object can be positive as well as negative; both signs are possible.
◯ True ◯ False

[1] 3. The entropy of an object can increase as well as decrease; both changes are possible.
◯ True ◯ False

[1] 4. In the real hydrogen atom, the orbital angular momentum $\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}}$ of the electron is not conserved.
◯ True ◯ False

[1] 5. In photoelectric effect, increasing the intensity of the light increases the current, but the value of the stopping voltage does not change.
◯ True ◯ False

[1] 6. The interaction energy of an electric dipole with an electric charge has a $1/r^3$ dependence on the distance r between them.
◯ True ◯ False

[1] 7. For a rigid body rotating in empty space, the direction of the angular velocity does not change.
◯ True ◯ False

[1] 8. An object flying through the air hits into the windshield of a rapidly moving bus. The force the windshield exerts on the object is smaller than the force that the object exerts on the windshield.
◯ True ◯ False

[1] 9. Although quantum mechanics allows the tunneling of particles through classically forbidden regions, when the position of the particle is measured, the probability of finding the particle in a classically

forbidden region will be exactly zero.

◯ True ◯ False

1 10. The wavelength of the photon decreases in the Compton scattering.

◯ True ◯ False

Part II: Classical Problems

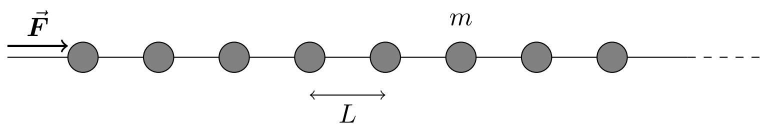
12 11. Explosions in underwater are different than the ones on the ground. Some water waves are emitted, which display some number of oscillations due to water bubbles formed. Consider an underwater nuclear explosion with the measured period of oscillations being τ. Below, you will find the energy E released in the explosion. Take ρ as the mass density of the water, p as the pressure, and d as the depth of the explosion.

(a) Use *dimensional analysis only* to obtain a formula for the energy E. Consider $E = E(\tau, \rho, p)$.

(b) Calculate E numerically if $\tau = 4 \times 10^{-3}\,\mathrm{s}$ and $d = 4000\,\mathrm{m}$. Take $g = 10\,\mathrm{m/s^2}$ and you know the density of water.

(c) Estimate the amount of mass converted to the energy in part (b).

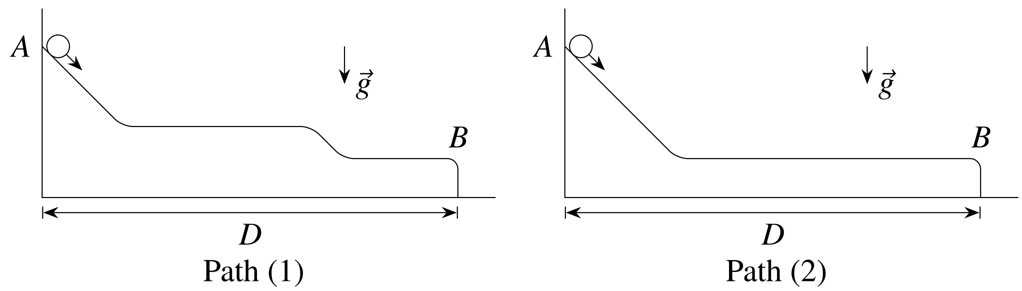
12 12. Many identical beads of mass m are located at equal distance L along a long smooth horizontal wire. Even though they are all at rest, the beads are allowed to slide freely on the wire. A constant force is now applied continuously to the leftmost bead, pushing it to the right. The effect of the force is transmitted among beads through collisions, which overall gives a wave propagating from left to right along the wire.



(a) If the collisions between beads are **perfectly elastic**, find the speed of the bead that the force F is applied as well as the speed of the front of the wave after a long time passes.

(b) If the collisions between beads are **completely inelastic**, find the speed of the bead that the force F is applied as well as the speed of the front of the wave after a long time passes.

10 13. There are two platforms, Path (1) and Path (2) which have identical heights at the left ends (point A, the starting point) as well as at the right ends (point B, the end point) as shown in the figure. The horizontal distance between points A and B are also the same and equal to D. However the shape of the path in between is different. The surfaces have identical properties like being smooth and no friction etc. Assume that there are two identical particles released at rest from the point A simultaneously and they remain in contact with the surface at all times.



(a) Who would win the race? Explain your reasoning clearly.

(b) Qualitatively sketch the shape of the path between A and B if one wants to make absolutely sure that such a path is always the winner with no exception.

8 14. Imagine one day people on the equator would start feeling weightless. What must have been changed about the Earth by then? Calculate numerically the rotation speed of the Earth as well as the length of a day.

16 15. Two identical relativistic particles of mass m, each with kinetic energy T, collide head-on in the lab frame. Consider the rest frame of one of the particles where the kinetic energy of the other particle is defined as T'.

(a) Calculate T' in terms of T and other parameters. Interpret each term physically.

(b) Numerically calculate relative kinetic energy T' and the relativistic factor γ' of the particle

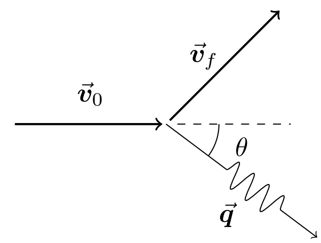
(i) if the particles are electrons with kinetic energy $T = 1$ GeV. What is the value of γ in the lab frame?
(Take the rest energy of the electron $m_e c^2 = 0.5 \times 10^{-3}$ GeV)

(ii) if the particles are protons with kinetic energy $T = 1$ GeV. What is the value of γ in the lab frame?

(Take the rest energy of the proton $m_p c^2 = 1$ GeV)

16 16. (a) Neutron scattering experiments are routinely used for determining dispersion relations of the phonons (sound quanta) of solids. For the purpose of this problem, phonons can be viewed as particles whose energy, ϵ, is related to their momentum, $\vec{\boldsymbol{q}}$, by the dispersion relation

$$\epsilon = s|\vec{\boldsymbol{q}}| = sq$$

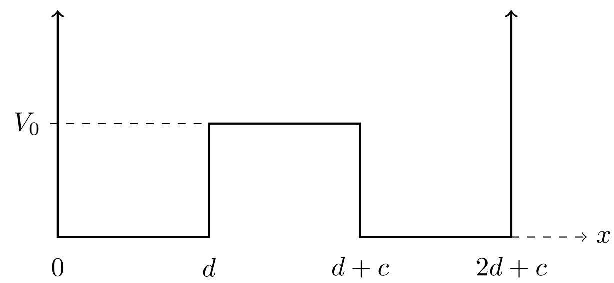


where s is the speed of sound in the solid. The neutrons move at non-relativistic speeds in these experiments.
Consider scattering events which involve creation of phonons. Neutrons with velocity $\vec{\boldsymbol{v}}_0$ are sent to a solid. Consider a neutron that produces a phonon at momentum $\vec{\boldsymbol{q}}$ and scatters off with a final velocity $\vec{\boldsymbol{v}}_f$.

(a) Show that there is a critical speed, s_c, such that if the incident neutrons move slower than this speed (i.e., if $v_0 < s_c$), they can never scatter by phonon creation. Such a neutron beam passes through the solid without any scattering. What is that critical speed?
Hint:First express final velocity $\vec{\boldsymbol{v}}_f$ *in terms of the phonon momentum,* $\vec{\boldsymbol{q}}$*. After that, use conservation of energy to find a relation between* q*,* θ *and* v_0*.*

(b) Show that there is a second critical speed, s_c', such that if the incident neutrons move with a speed between these two critical values, (i.e., if $s_c < v_0 < s_c'$), then they produce phonons only inside a cone, i.e., θ takes values between 0 and some maximum value $\theta_{\max}$. Find s_c' and $\theta_{\max}$.

16 17. Consider a particle in one-dimensional space. The potential energy profile as a function of x is drawn below where V_0, d, and c are parameters of the problem. Assume that energies of first two levels are smaller than V_0 (i.e., $E_0 < E_1 < V_0$).



(a) Sketch roughly the wavefunctions for the first two energy eigenstates.
Hint: You may shift the origin.

(b) Take the limit $c \to \infty$. Then show that the two lowest energy eigenvalues would both go to the ground state of the given potential with $c \to \infty$.

1.2 Misc-May-2018

Part I: True/False and Multiple-Choice Questions

Choose True or False for the statements below. In **either case**, write **one** short but **meaningful** sentence underneath to explain why. If you don't, you will not get the point for that statement.

1 18. An object flying through the air hits into the windshield of a rapidly moving car. The force the windshield exerts on the object is higher than the force that the object exerts on the windshield.
◯ True ◯ False

1 19. As Galilean relativity is considered as an approximation of special relativity valid for low speeds, special relativity is considered as an approximation of general relativity valid for weak gravitational fields.
◯ True ◯ False

[1] 20. Orbital angular momentum of Hydrogen atom in the ground state is zero.
◯ True ◯ False

[1] 21. The motion of the CM of an object depends only on the total external force acting on the object and not on whether it is rotating or not.
◯ True ◯ False

[1] 22. Fine-structure splitting is due to the interaction of the spin of the nucleus with the magnetic field of the atom.
◯ True ◯ False

[1] 23. A photon cannot decay to an electron and a positron because mass is not conserved.
◯ True ◯ False

[1] 24. Observers who are in motion with respect to each other may disagree on whether two events occurred at the same time or one occurred before the other.
◯ True ◯ False

[1] 25. The frequency of light incident from air to glass decreases as it is transmitted into the glass.
◯ True ◯ False

[1] 26. There exist a field in the low index medium in the case of total internal reflection at a boundary between a high index and a low index medium.
◯ True ◯ False

[1] 27. In order for total internal reflection to take place, light must be incident from a low index medium on to a high index medium at an angle larger than the critical angle.
◯ True ◯ False

[1] 28. When a plane wave passes through an aperture with a size close to the wavelength, the angular spread of the diffracted beam increases when the diameter of the aperture increases.
◯ True ◯ False

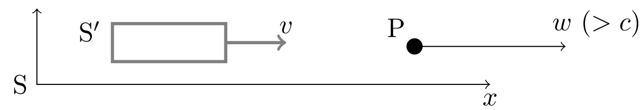
1 29. The angular spread of a diffracted beam from an aperture does not depend on the wavelength.
◯ True ◯ False

Part II: Classical Problems

15 30. Stefan-Boltzmann law relates the power per unit area, S, radiated by a black-body to its temperature T. Stefan discovered the law by experiments in 1879, and Boltzmann derived the power of temperature by using some thermodynamics ideas in 1884. However, nobody at that time could theoretically derive the law because they didn't know about quantum physics.

Amusingly, by using the fact that quantum physics *has something to do with it*, it is possible to derive the law without using any physics at all. This is what you know: S depends on T and the S vs T relation also involves the Planck constant, h, the speed of light, c, and Boltzmann's constant k_B. No other fundamental constants enter into the relation. In other words, $S = f(T, h, c, k_B)$ for some function f. Using dimensional analysis (i.e., using the fact that such a relation is dimensionally correct), show that $S \propto T^n$ and find n.

15 31. Using special relativity theory, it is possible to show that no object or signal can move faster than the speed of light, c. In this problem, you will see what the problem is. Suppose that there is a *superluminal* particle P which moves with velocity $w > c$ along the $+x$-direction relative to a frame S. Consider another frame S′ which moves with velocity v relative to S.



(a) Show that there is a critical velocity v_c of the frame S′, which is subluminal (i.e., $|v_c| < c$), such that P moves with *infinite* velocity relative to S′. What is that critical velocity?
(*Hint: Use Lornetz transformations and express $\Delta x'/\Delta t'$ for particle P.*)

(b) Show that, if the frame S′ moves faster than this critical velocity ($v > v_c$), then P *goes backwards in time* relative to S′. In other words, show that $\Delta t' < 0$ even though $\Delta t > 0$.

18 32. Neutral hydrogen is one of the most abundant element in the early universe. The interaction of the spin of the electron with the spin of the proton is responsible for the splitting in the ground state of the hydrogen, a phenomenon known as the hyperfine splitting. Studying the radiation (in the form of absorption or emission) arising from the transitions between these splitted states provides information about the density of the neutral hydrogen gas in the interstellar medium and could even reveal evidence about the formation of first starts at cosmic dawn. Recently the EDGES (Experiment to Detect the Global EoR Signature) collaboration announced, for the first time, the observation of an absorption line from atomic hydrogen which is something stronger than any known prediction. Thus, it is interpreted as radiation from the first stars. If $\boldsymbol{S}_e$ and $\boldsymbol{S}_p$ are the spin operators of the electron and proton, respectively. Then one can define magnetic dipole operator for, say, proton $\boldsymbol{M}_p = \frac{e\, g_p}{2m_p}\boldsymbol{S}_p$. Here $g_p = 5.56$ is the proton gyromagnetic ratio. The magnetic field created by $\boldsymbol{M}_p$ at the origin is $\boldsymbol{B} = \frac{2\mu_0}{3}\delta(\vec{\boldsymbol{r}})\vec{\boldsymbol{M}}_p + \ldots$. Here only the part of $\vec{\boldsymbol{B}}$ with a non-zero contribution to the energy is shown.

(a) Obtain the interaction Hamiltonian and compute the energy associated with this interaction for the ground state of the hydrogen.
Hint: The radial wave function at the origin is $|R_{10}(0)|^2 = 4\left(\frac{\alpha m_e c}{\hbar}\right)^3$

(b) Find the possible total spin states of the system and using part (a) to obtain the hyperfine energy.

(c) Compute energy numerically and determine which state is more populated. Is transition possible? Compute also the wavelength of the photon emitted/absorbed? Which part of the electromagnetic spectrum does it belong to?

10 33. Consider the following Kepler orbits around the Sun to calculate the eccentricity and the orbital period. *Hint: You may use the following. The semimajor axis of the Earth's orbit* $= 1\,AU = 1.496 \times 10^{11} m$ *and the Earth's orbital period = 365.25 d. The orbit equation is* $\frac{\alpha}{r} = 1 + \epsilon \cos\theta$.

(a) An orbit with perihelion distance (closest to the Sun) 1.8 AU and aphelion distance (farthest from the Sun) 2.8 AU.

(b) An orbit with aphelion distance 1.6 AU with an orbital speed 16 km/s.

15 34. A circular laser beam of $10.0\,mW$ power with $532.0\,nm$ wavelength and $10.00\,mm^2$ cross-sectional area is normally incident on a piece of clear flat glass surface ($n = 1.480$) surface from air.

(a) Calculate the E-field amplitude of the reflected beam from the water surface. ($\epsilon_0 = 8.850 \times 10^{-12} F/m$)

(b) What is the wavelength of the transmitted beam?

(c) What is the phase velocity of the transmitted beam?

(d) What is the frequency of the transmitted beam?

15 35. A beam of light with wavelength of 1.00 μm and $M^2 = 20$ passes through an aperture of diameter 2.0mm.

(a) Find the divergence angle of the beam in degrees (give the cone full-angle).

(b) Calculate the diameter of the beam at a distance of 5.00 m away from the aperture in units of cm.

(c) If the longitudinal (temporal) coherence length is 80 times the transverse (spatial) coherence length, what is the wavelength linewidth of the light in units of pm?

1.3 Misc-Nov-2017

Part I: True/False and Multiple-Choice Questions

Choose True or False for the statements below. In **either case**, write **one** short but **meaningful** sentence underneath to explain why. If you don't, you will not get the point for that statement.

1 36. Gravitational force does work on a satellite which is in a circular orbit around the Earth.
◯ True ◯ False

1 37. Total energy does not change under Lorentz transformations.
◯ True ◯ False

1 38. Electric dipole moment of a point charge placed away from the origin is not zero.
◯ True ◯ False

1 39. Expectation value of an observable in an energy eigenstate does not change in time.
◯ True ◯ False

1 40. The *wavelength* of light incident from glass to air *increases* as it is transmitted into the air.
◯ True ◯ False

1 41. The *frequency* of light incident from glass to air *decreases* as it is transmitted into the air.
◯ True ◯ False

1 42. In order for total internal reflection to take place, light must be incident from a low index medium on to a high index medium at an angle larger than the critical angle.

◯ True ◯ False

1 43. When a plane wave passes through an aperture with a size close to the wavelength, the transmitted beam shows angular spreading pattern.
◯ True ◯ False

1 44. An ideal gas in *free expansion* in a thermally isolated chamber undergoes an increase in its internal energy due to the increased volume available to the molecules.
◯ True ◯ False

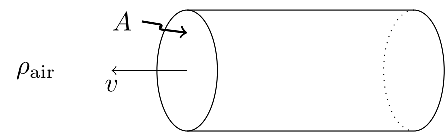
1 45. The entropy increases monotonically as a function of internal energy for all systems in equilibrium.
◯ True ◯ False

1 46. Since a black body is a perfect absorber, it does not emit any radiation at any temperature.
◯ True ◯ False

1 47. For systems whose entropy is bounded (such as spin systems), the entropy decreases with increasing energy after a certain point. This is how we get negative temperatures.
◯ True ◯ False

Part II: Classical Problems

12 48. Air friction is the dominant energy loss mechanism for cars moving at high speeds. As a result, almost all power used by fast cars is spent on overcoming the air friction forces. To find the power P needed, we simplify the problem by modeling the car as a cylindrical object. Using this model, it can be seen that P essentially depends on three quantities.



One of these quantities is the density of air, ρ_{air}, because the air molecules in the car's path must be pushed away and their inertia is the cause of the frictional forces. The other quantity is the cross-sectional area, A, of the car, because this area determines how much air must be pushed away. The length of the car should therefore be unimportant. Finally, P must depend on the speed, v, of the car. As a result, power needed must be a simple function of these three quantities, $P = f(A, \rho_{\text{air}}, v)$.

(a) Find this relationship by using *dimensional analysis* (i.e., find $P = f(A, \rho_{\text{air}}, v)$ by using only the fact that the units come out correctly.)

(b) Suppose that a car increases its speed from 70 km/h to 140 km/h. How much more power should then be necessary to be used?

8 49. There is an urban legend claiming that the direction of draining water in washbowls are different in each hemisphere and this is associated with the effect of the Coriolis force. Make a rough estimation of the effect to confirm or refute the legend. Take $\omega_{\text{Earth}} \simeq 7 \times 10^{-5}$ rad/s.

12 50. A particle with mass m_0 disintegrates into two identical particles, each with mass m. One of the product particles leaves with a speed of $3c/5$ along the $+x$-direction. Parent particle was at rest before the disintegration. Write down the momentum 4-vectors of each particle and by using the conservation laws

(a) find the velocity of the other product particle.

(b) Find the m_0/m ratio.

12 51. Consider a particle with mass m in a one-dimensional box of length L (suppose box is given as $0 \le x \le L$). Let this particle be in the nth energy level. We are interested in the force, F_n, applied by this particle on the *right wall*, i.e., the wall at $x = L$. There are two possible ways of computing this force.

(a) We can treat the particle classically, i.e., think of it as a classical particle having energy E_n. This particle hits the right wall periodically. At each collision, it imparts an impulse on the wall, and from the impulse per unit time, we can find the *average* force between the wall and the particle. Compute the following: (i) the speed, v_n, of the particle, (ii) the period, T_n, between the collisions with the right wall, (iii) momentum change, Δp_n, of the particle in each collision, and finally (iv) the average force on the particle. This force is also equal to the force on the wall.

(b) An alternative method uses energy concept. Suppose that the right wall has moved towards right by a small distance ΔL. The work done on the wall by the particle, which is the same as the energy transferred from the particle to the wall, is equal to the energy loss of the particle. Using this principle, express the force F_n as a derivative and compute it. Compare with the semiclassical result obtained above.

12 52. A circular laser beam of 4.0 mW power with 632.8 nm wavelength and 12.00 mm^2 cross-sectional area is normally incident on a still water ($n = 1.330$) surface from air.

(a) Calculate the E-field amplitude of the reflected beam from the water surface. ($\epsilon_0 = 8.850 \times 10^{-12}$ F/m)

(b) What is the wavelength of the transmitted beam?

(c) What is the phase velocity of the transmitted beam?

(d) What is the frequency of the transmitted beam?

12 53. An LED (light-emitting diode) has an emitting surface of 1.0 mm diameter. Light power of 6.0 mW is collected by a lens with 24 mm focal length and 18 mm diameter, placed at a 120 mm distance from the LED.

(a) Determine the position of the LED image.

(b) Determine the diameter of the LED image.

(c) Determine the intensity of the LED image.

10 54. The bandgap energy of $Al_xGa_{1-x}As$ material has a dependency on the Al fraction x as follows: $E_g = 1.424 + 1.427x + 0.041x^2$ (eV). A layer of an unknown composition $Al_xGa_{1-x}As$ sample is illuminated with light of variable wavelength, and it is found that the transmission through the layer decreases rapidly for wavelengths shorter than 726.5 nm.

(a) Determine the Al fraction x in this particular compound.

(b) Determine the energy bandgap of $Al_xGa_{1-x}As$ at this composition.

10 55. In a rotating galaxy, let $v(r)$ denote the speed of stars which are at a distance r from the galactic center and $\rho(r)$ denote the mass density at this distance. Observational measurements indicate that $v(r)$ is (almost) independent of r after certain point. Based on this information, what can you say about the r dependence of $\rho(r)$? For simplicity, assume that the galaxy is spherically symmetric.
Note: Observations of this kind, which are first done in the seventies, led to the idea of dark matter.

1.4 Misc-May-2017

Part I: True/False and Multiple-Choice Questions

Each correct answer is worth +1 point. Each wrong answer is worth -1 point.

1 56. The de Broglie wavelength of an electron gives an indication of its size.
◯ True ◯ False

1 57. When a blackbody's temperature is doubled, its energy output per unit time is also doubled.
◯ True ◯ False

1 58. For an electron in $\ell = 3$ in a hydrogen atom, there are 6 possible values (with respect to z axis) for the direction of angular momentum.
◯ True ◯ False

1 59. A magnetic dipole in a uniform magnetic field will feel a net force (when its orientation is appropriate).
◯ True ◯ False

1 60. Entropy is an extensive quantity.
◯ True ◯ False

1 61. The escape speed from Earth is around 2 km/s.
◯ True ◯ False

1 62. The wavefunctions of bosons are completely anti-symmetric under particle exchange.
◯ True ◯ False

1 63. A dielectric material responds to an externally applied electric field in a way to decrease it inside the material.
◯ True ◯ False

1 64. In relativity time-order of some events may be switched. For example, if event-1 happens before event-2 in one frame, it is possible that event-1 may happen later than event-2 in some other frame.
◯ True ◯ False

1 65. When the current that pass over an inductor decreases, the emf generated in the inductor is in the same direction as the current.
◯ True ◯ False

1 66. The general solution of an nth order ordinary differential equation of a single variable depends on n arbitrary parameters.
◯ True ◯ False

1 67. Although neutrons are charge neutral, it is possible to produce an electric field by making them move.
◯ True ◯ False

Part II: Classical Problems

12 68. Consider a one solar mass Schwarzschild black hole with the Schwarzchild radius $r_s = \frac{2GM_\odot}{c^2}$ and the horizon area $A_s = 4\pi r_s^2$. The Hawking temperature of the black hole is given as $T_H = \frac{\hbar c^3}{8\pi G M_\odot k_\beta}$. Considering the black hole as a perfect blackbody and using the Stefan-Boltzmann law, estimate the time that will take for the black hole to evaporate, i.e. radiate all of its mass into energy. How does your answer compare with current age of the universe, which is roughly 14 billion years?

Recall that Stefan-Boltzmann constant is $\sigma = \frac{\pi^2 k_\beta^2}{60\hbar^3 c^2} = 5.67\times10^{-8} W\cdot m^{-2}\cdot K^{-4}$ *and* $M_\odot = 1.98\times10^{30} kg$.

10 69. When objects like comets, asteroids, or meteors hit the surface of the Earth, large holes in the shape of circles are formed, known as *impact craters*, one of such is shown in the figure. In the history of the Earth, one of the biggest of them, known as the Kara crater, was assumed to be formed around 70 million years ago in the Yugorsky Peninsula, Russia. The diameter of the Kara crater is estimated to be around 120 km at the time of the impact. We are curious about the energy E of the object giving the Kara crater. Using dimensional analysis, show that the energy $E \propto d^4$.

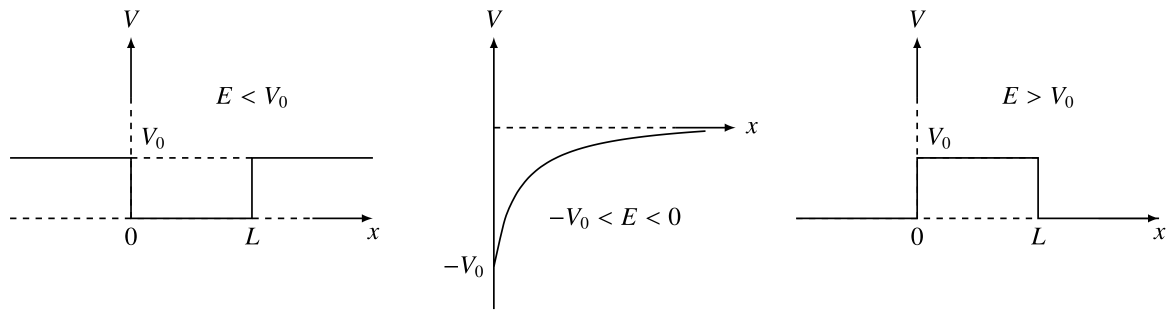
Hint: You may assume that the diameter $d = d(E, g, \rho)$ *where* g *is the gravitational acceleration and* ρ *is the density of the soil.*

12 70. Consider the one-dimensional potentials given below for three different systems. For each of these potentials, put your answers with explanations to the following questions next to each figure.

(i) State the boundary condition for the wavefunction $\psi(x)$ and its derivative $\psi'(x)$,

(ii) State whether the energy is expected to be continuous or quantized, or both.

(iii) Determine whether each system possesses any symmetry leading to separations of the odd and even solutions.



15 71. Consider a particle of mass m in a one-dimensional box

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ \infty & \text{otherwise,} \end{cases}$$

Let us justify the so-called correspondence principle in the given system.

(a) Treat the problem classically first and calculate the probability for the particle to be in the interval, say, $(0, L/4)$.

(b) Now consider the quantum mechanical probability density in the nth state. What is the probability for the particle to be in the same interval, $(0, L/4)$.

(c) Take large n limit of the probability you got in part (b). Compare with your result in part (a).

12 72. Consider that this exam is given by an instructor who happens to be in a spaceship moving away from Earth with normalized relativistic speed $\beta = \frac{v}{c}$. Here c is the speed of light.

The exam starts on the Earth exactly at the time when the spaceship is passing by the Earth. The exam is scheduled to be over in T_0 hours in the instructor's clock. Hence, the instructor sends a light signal back to the class when his clock reads the elapsed time as T_0 and the students stop writing when the signal reached them.

(a) Find an expression for the time the students use for the exam (in terms of β and T_0)?

(b) If $T_0 = 1$ hour and $\beta = 0.96$, calculate the duration that the students use on the Earth.

15 73. A proton that is restricted to move between $x = 0$ and $x = L$, has an energy of 500 keV in the first excited state.

(a) What is the width L?

(b) What will be the wavelength of the photon that is emitted when the proton jumps to the ground state?

(c) How does the probability of finding the proton in the vicinity of $x = L/2$ change when it makes this jump? Explain your answer, by using sketches if necessary.

12 74. Which of the following wavefunction(s) have a definite momentum? Find the eigenvalue for that one(s); choose one of the others and explain why it does not have a definite momentum. (A, B, C, k_1, k_2, k_3 are constants.)

(a) $A \sin k_1 x$
(b) $B(\sin k_2 x + \cos k_2 x)$
(c) $C(\sin k_3 x + i \cos k_3 x)$

1.5 Misc-Nov-2016

Part I: True/False and Multiple-Choice Questions

Each correct answer is worth +1 point. Each wrong answer is worth -1 point.

[1] 75. In Rayleigh scattering the wavelength of the scattered photon changes.
◯ True ◯ False

[1] 76. Fine-structure splitting is due to the interaction of the magnetic moment of the nucleus with the magnetic field of the atom.
◯ True ◯ False

[1] 77. In a non-dispersive medium the group velocity will be equal to the phase velocity.
◯ True ◯ False

[1] 78. The magnetic field of the Earth has the same value in Ankara as it does at the North Pole.
◯ True ◯ False

[1] 79. The size of an object is a good estimate of the uncertainty in its position.
◯ True ◯ False

[1] 80. When a blackbody's temperature is doubled, its energy output per unit time is also doubled.
◯ True ◯ False

[1] 81. When an electron and a proton are each accelerated from rest through the same potential difference, they will have the same de Broglie wavelength.
◯ True ◯ False

[1] 82. The frequency of the emitted photon is higher in stimulated emission than in spontaneous emission.
◯ True ◯ False

[1] 83. For *two dimensional* particle in a box, we can have at most 2 electrons in the first energy level above ground state.
◯ True ◯ False

[1] 84. Any two solutions of time dependent Schrödinger's equation can be added together to give another possible solution to this equation.
◯ True ◯ False

[1] 85. Any two solutions of time independent Schrödinger's equation can be added together to give another possible solution to this equation.
◯ True ◯ False

Part II: Classical Problems

[12] 86. (a) Assume that your car of mass m and velocity $\boldsymbol{v}$ collides
(i) to a hard wall.
(ii) to another car of mass $2m$ with a velocity $-\frac{\boldsymbol{v}}{2}$.
(iii) to another car of mass $m/2$ with a velocity $-2\boldsymbol{v}$.

After the collision explain whether your car keeps moving or not? Explain clearly in which case your car would get the most damage. Then decide which option(s) appears to be safer under the circumstances.

(b) A car and a large truck collide head on and stick together.

(i) Explain clearly what happens in regard to the change of momentum.

(ii) Explain clearly which one experiences larger acceleration during the collision.

(c) Assume that a incident object hits a stationary one. Would it be possible for the stationary one to have a larger momentum than the incident one? If not explain why. If so, give a clear example.

16 87. (a) Utku is on one end of a train of length L, moving with respect to a platform, on which Toprak is at rest. Utku observes two events, one taking place at the middle of the train at time $t_1 = L/c$ and another taking place at the far end of the train at time $t_2 = 3L/4c$. Toprak observes these two events simultaneously.

(i) What is the speed of the train with respect to the platform?

(ii) Is Utku at the front or the rear end of the train? (The train is moving in the forward direction.) *Explain your answer, no credit will be given without a correct explanation.* *Note: This part can be answered independent of the previous part, but you can use your result from previous part if you want.*

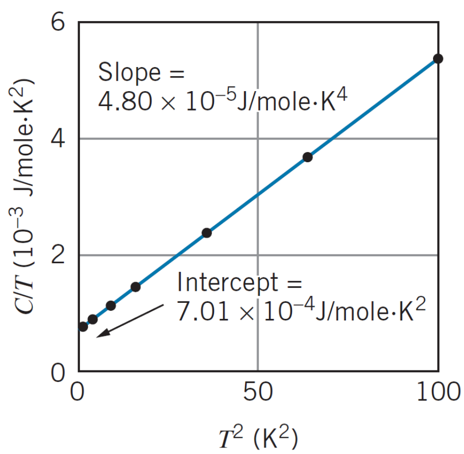
(b) A system of 4 identical particles are sharing 5 units of energy. What is the probability of a particle having 2 units of energy, if

(i) the particles are distinguishable.

(ii) the particles are indistinguishable with spin 1/2.

(iii) the particles are indistinguishable with spin 1.

12 88.



The C/T as a function of T^2 for Cu is shown in the figure.

(a) What is the functional form of molar heat capacity for Cu? Write C as a power series in T, leaving the coefficients undetermined.

(b) What do the terms in this series correspond to? What are the contributing factors?

(c) From the values in the graph, determine the values of these coefficients.

(d) What is the Debye temperature for Cu?

Hint: According to Debye, the lattice contribution to heat capacity at low temperatures is given by

$$C = \frac{12\pi^4}{5} R \left(\frac{T}{T_D}\right)^3 .$$

12 89. Consider a particle in an infinite potential well

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ \infty & \text{otherwise,} \end{cases}$$

with energy levels E_n; and another one in finite potential well

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ U_0 & \text{otherwise,} \end{cases}$$

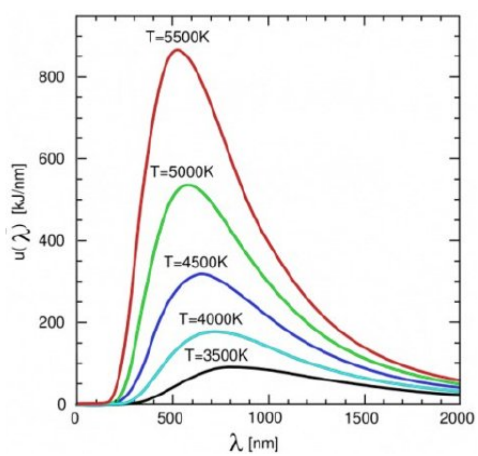
with energy levels E'_n, where $U_0 > 0$.

(a) Sketch the first excited state wave functions for these particles.

(b) Using these sketches, argue which particle would have the higher energy.

(c) Using this result, argue which particle would have a higher uncertainty in momentum. Discuss this result's compatibility with the uncertainty principle.

10 90. When an object is heated up it can be assumed to emit radiation as a perfect blackbody if its emissivity is equivalent to 1. Using the given graph answer the following questions:



(a) In the Rayleigh-Jeans formula the energy density of cavity radiation is given by

$$u(\nu) = \frac{8\pi n^3 \nu^2}{c^3} kT \ ,$$

where ν is the frequency, n is the refractive index and k is the Boltzmann constant. In which part of the spectrum shown in the graph does this formula fail? Explain clearly.

(b) Explain how Planck overcame this dilemma using one or two sentences.

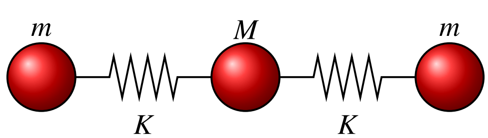
(c) Assuming our Sun is yellow, which curve and corresponding temperature in the above graph is closest to its actual temperature?

15 91. Consider a linear system comprising one particle with mass M and two particles with mass m on either side as shown below (a model for the carbon dioxide molecule). The particles are connected with identical springs with spring constant K (Do not use energy formalism to solve the following problem).

(a) Considering only the motion of the masses along the line connecting them, write down the equation of motion for each particle.

(b) Find the eigenvalues (normal frequencies): Show that $\omega = 0$ is a normal mode of this system. Find the two remaining non-zero normal mode frequencies.

(c) What are the ratios of the amplitudes of the three masses? What kind of motion does the $\omega = 0$ solution correspond to?



12 92. A CW diode laser has a power output of P_0 at a center wavelength of λ_0. If the beam is circular with a diameter of d_0, using the constants c, ϵ_0, μ_0 for speed of light, permittivity of free space and magnetic permeability and vector quantities $\boldsymbol{E}_0, \boldsymbol{B}_0$ for the Electric Field and Magnetic Field Strength respectively,

(a) Formulate the intensity of the laser beam if traveling through free space

(b) Formulate the intensity if the beam is traveling inside a slab of glass with index of refraction n.

(c) If the beam has a Gaussian TEM_{00} profile, formulate the intensity of the beam at a distance equivalent to the beam waist (ω_0) from the optical axis?

(d) If the bandwidth of this laser is $\delta\nu$ (in units of Hz). Formulate the coherence length of this laser source.