Local Realism, Non-Contextuality and Cloning

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Bell-Clauser-Horne-Shimony-Holt

a

b ± 1 b'

a'

Bell-Clauser-Horne-Shimony-Holt

$$S \equiv ab + ab' + a'b - a'b'$$

$$= (a + a')\, b + (a - a')\, b'$$

$a = a'$ → $2ab$

$a \neq a'$ → $2ab'$

$$|S| \leq 2$$

Bell-Clauser-Horne-Shimony-Holt

Quantum

$$S = \langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle$$

$$\downarrow$$

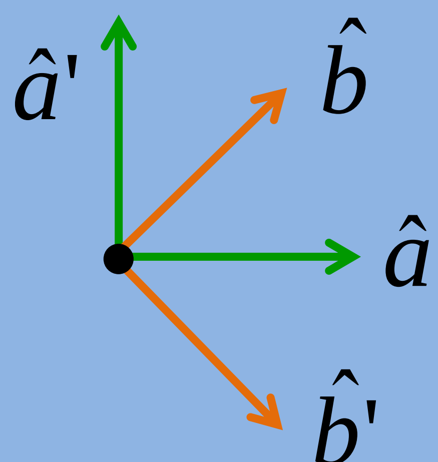
$$2\sqrt{2}$$

Bell-Clauser-Horne-Shimony-Holt

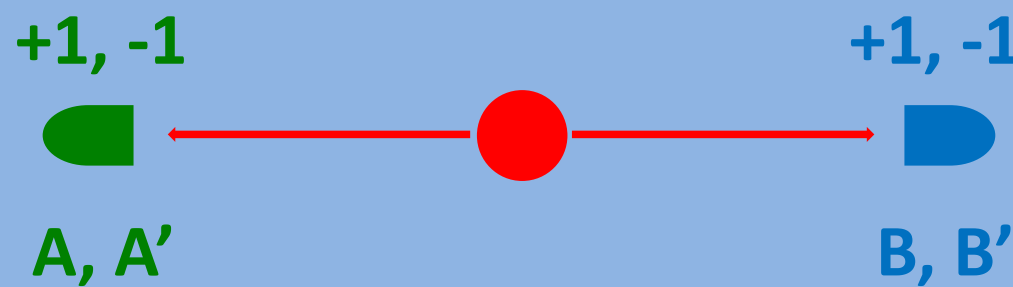
Quantum

$$\left|\Psi\right\rangle = \frac{\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle}{\sqrt{2}}$$

$$A = \vec{\sigma}\cdot\hat{a},\quad A' = \vec{\sigma}\cdot\hat{a}',\quad B = \vec{\sigma}\cdot\hat{b},\quad B' = \vec{\sigma}\cdot\hat{b}'$$



Bell-Clauser-Horne-Shimony-Holt



$$E_{AB} = P_{AB}(=) - P_{AB}(\neq) = \langle AB \rangle$$

$$S = E_{AB} + E_{AB'} + E_{A'B} - E_{A'B'}$$

$$|S| \leq 2$$

Local Realism

Klyachko-Can-Binicioğlu-Shumovsky

a_1

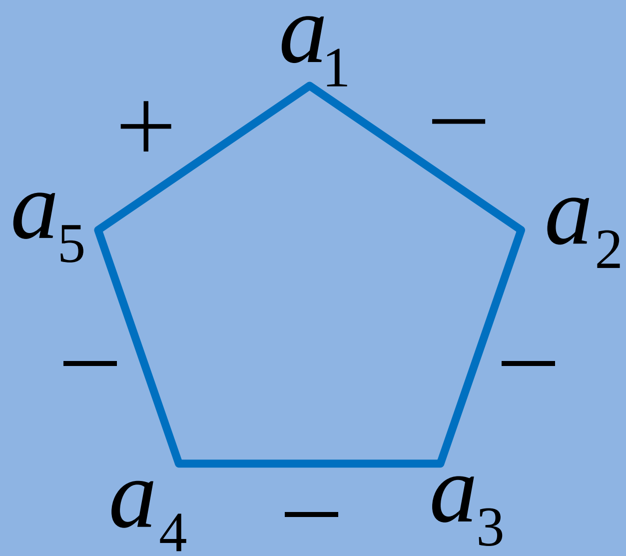
a_5 a_2

± 1

a_4 a_3

Klyachko-Can-Binicioğlu-Shumovsky

$$S \equiv a_1 a_2 + a_2 a_3 + a_3 a_4 + a_4 a_5 + a_5 a_1$$



$$S \geq -3$$

Klyachko-Can-Binicioğlu-Shumovsky

Quantum

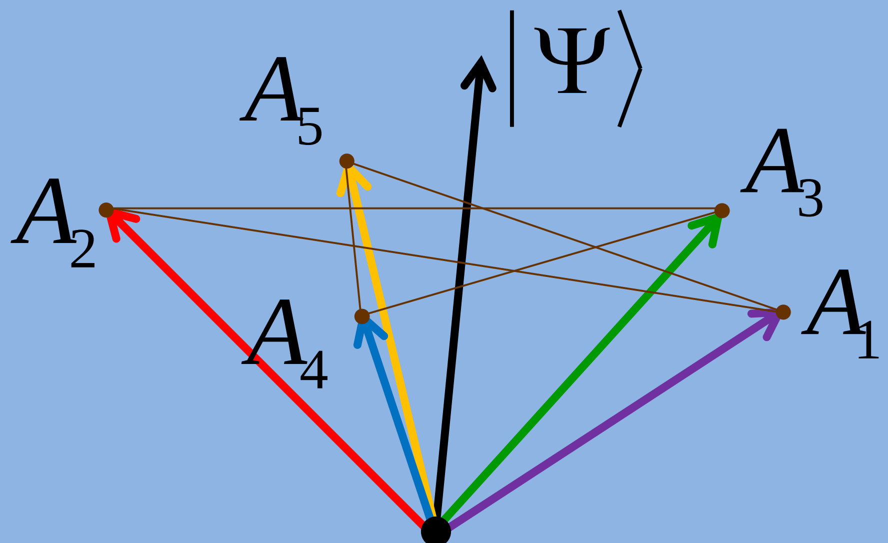
$$S = \langle A_1 A_2 \rangle + \langle A_2 A_3 \rangle + \langle A_3 A_4 \rangle + \langle A_4 A_5 \rangle + \langle A_5 A_1 \rangle$$

$$\downarrow$$

$$5 - 4\sqrt{5}$$

Klyachko-Can-Binicioğlu-Shumovsky

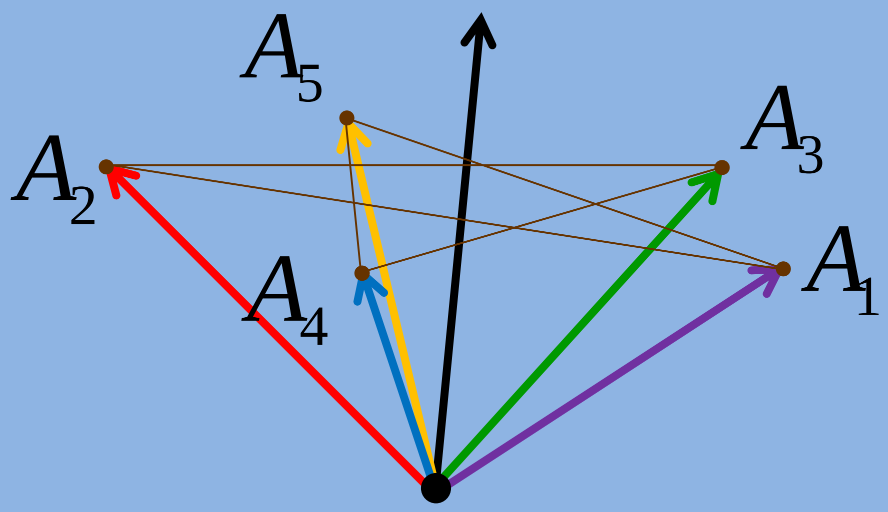
Quantum



$$A_i = 2S_i^{\,2} - I$$

spin-1

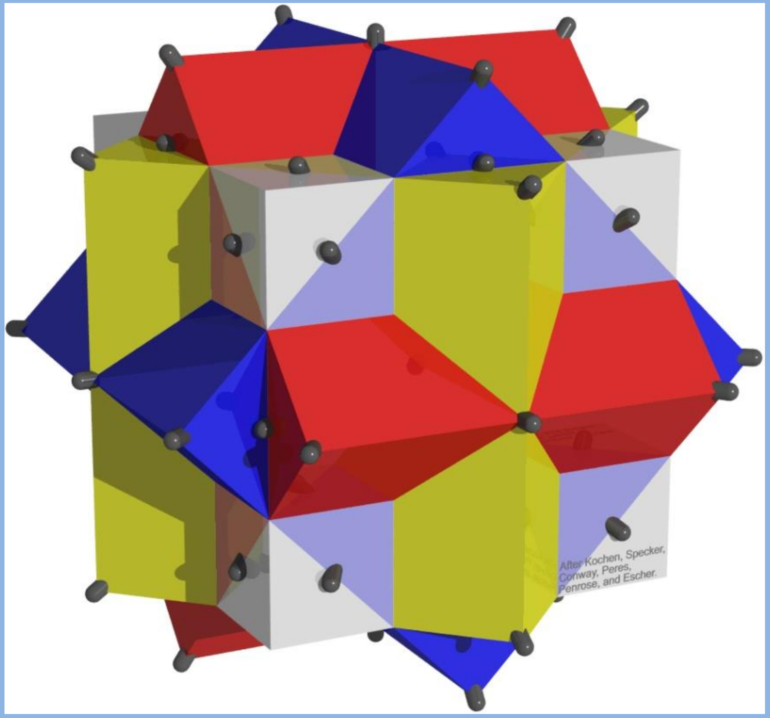
Klyachko-Can-Binicioğlu-Shumovsky



Non-Contextuality

Kochen-Specker Theorem

Quantum mechanics is contextual.
Results of quantum mechanics cannot be fully explained by non-contextual theories which assume that the measurement outcomes of a physical system are *predetermined* and independent of their own and other simultaneous compatible measurements.



Monogamy of CHSH and KCBS Inequalities

$$\left|S_{CHSH}^{AB}\right| + \left|S_{CHSH}^{AC}\right| \leq 4$$

$$S_{CHSH}^{AB} + S_{KCBS}^{A} \geq -5$$

Filiz

FE **FE** **FE** **FE**

$$\frac{|00\rangle\langle 00| + |11\rangle\langle 11|}{2} = \frac{|{+}{+}\rangle\langle {+}{+}| + |{-}{-}\rangle\langle {-}{-}|}{2}$$

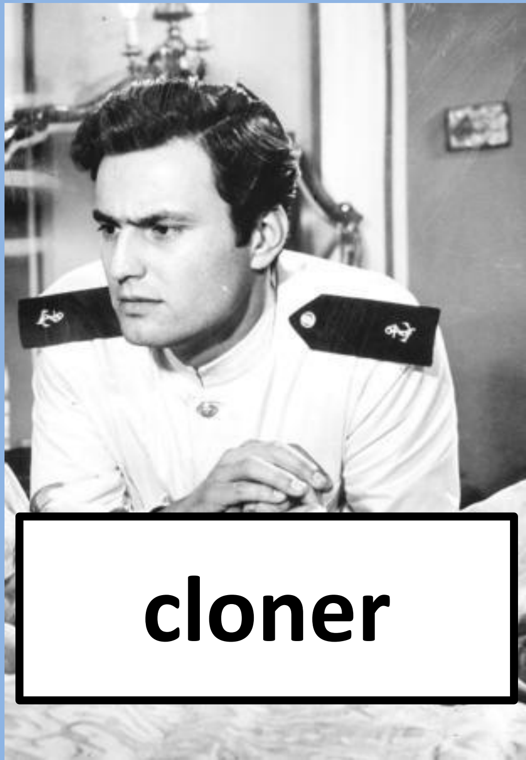
E **E** **E** **Ediz**

$$|0\rangle \rightarrow |0\rangle \otimes \ldots \otimes |0\rangle$$

$$|1\rangle \rightarrow |1\rangle \otimes \ldots \otimes |1\rangle$$

$$|+\rangle \rightarrow |+\rangle \otimes \ldots \otimes |+\rangle$$

$$|-\rangle \rightarrow |-\rangle \otimes \ldots \otimes |-\rangle$$



special relativity (1905)

The speed of light in vacuum is the maximum speed at which all energy, matter, and information in the universe can travel.

cloning 0

$$|0\rangle|0\rangle|0\rangle$$

$$\downarrow$$

$$\sqrt{\frac{2}{3}}|0\rangle|0\rangle|0\rangle+\sqrt{\frac{1}{6}}\Big(|0\rangle|1\rangle+|1\rangle|0\rangle\Big)|1\rangle$$

cloning 1

$$|1\rangle|0\rangle|0\rangle$$

$$\downarrow$$

$$\sqrt{\frac{2}{3}}|1\rangle|1\rangle|1\rangle+\sqrt{\frac{1}{6}}\Big(|1\rangle|0\rangle+|0\rangle|1\rangle\Big)|0\rangle$$

$$|\psi\rangle|0\rangle|0\rangle$$

$$\downarrow$$

$$\sqrt{\frac{2}{3}}|\psi\rangle|\psi\rangle|\psi^{*}\rangle+\sqrt{\frac{1}{6}}\left(|\psi\rangle|\psi^{\perp}\rangle+|\psi^{\perp}\rangle|\psi\rangle\right)|\psi^{*\perp}\rangle$$

$$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$$

$$|\psi^{\perp}\rangle=\beta^{*}|0\rangle-\alpha^{*}|1\rangle$$

$$|\psi\rangle=\alpha^{*}|0\rangle+\beta^{*}|1\rangle$$

fidelity

$$\rho_A = \frac{5}{6}\left|\psi\right\rangle\left\langle\psi\right| + \frac{1}{6}\left|\psi^{\perp}\right\rangle\left\langle\psi^{\perp}\right|$$

$$F_A = \left\langle\psi\left|\rho_A\right|\psi\right\rangle = \frac{5}{6}$$

Gisin and Massar

$$1 \to M$$

$$F = \frac{2M + 1}{3M}$$

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Quantum cloning without signaling

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Abstract

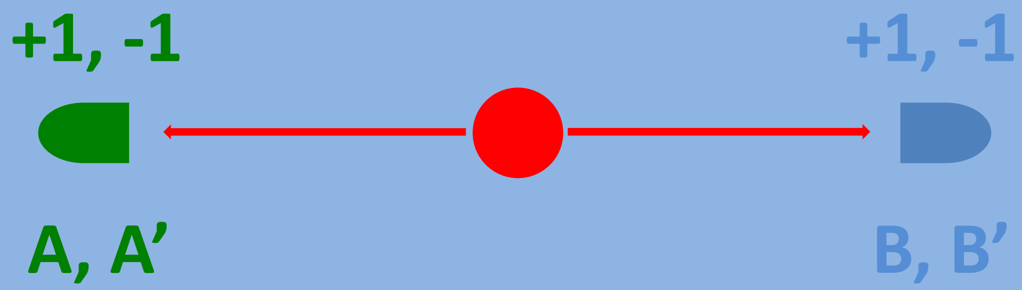
Perfect quantum cloning machines (QCM) would allow one to use quantum non-locality for arbitrary fast signaling. However, perfect QCM cannot exist. We derive a bound on the fidelity of QCM compatible with the no-signaling constraint. This bound equals the fidelity of the Bužek–Hillery QCM.

$$1 \rightarrow 2$$

Nicolas Gisin

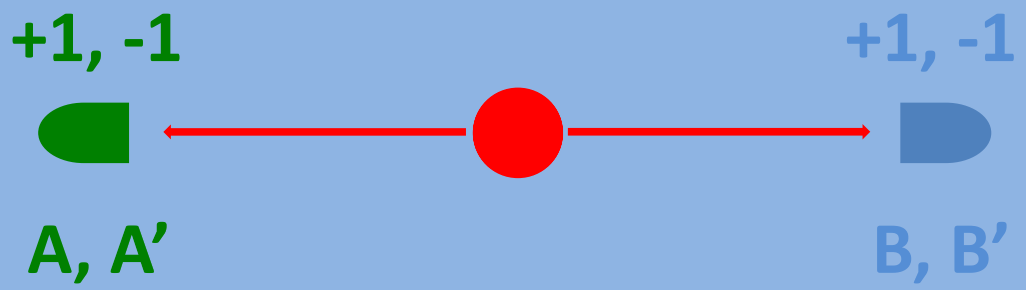
$$\left(F_{M=2}^{NS}\right)_{\max} = \frac{5}{6} = \left(F_{M=2}^{Q}\right)_{\max}$$

A quite straightforward proof of the optimality of the Bužek–Hillery QCM [3] has been presented, based on the fact that no quantum process can provide arbitrarily fast signaling [7]. Once again, quantum mechanics is right at the border line of contradicting relativity, but does not cross it. The peaceful coexistence between quantum mechanics and relativity [1] is thus re-enforced. It is intriguing that the no signaling constraint is a powerful guide to find the limits of quantum mechanics [8].



$$P_{AB}(1,1)+P_{AB}(1,-1)=P_{AB}(1,?)=P_{AB'}(1,1)+P_{AB'}(1,-1)$$

no-signaling



$$E_{AB} = P_{AB}(=) - P_{AB}(\neq) = \langle AB \rangle$$

$$S = E_{AB} + E_{AB'} + E_{A'B} - E_{A'B'}$$

$$|S| \leq 2$$

$$2 \le |S| \le 2\sqrt{2}$$

superquantum nonlocal correlations

$$\downarrow$$

stronger violation of Bell inequalities

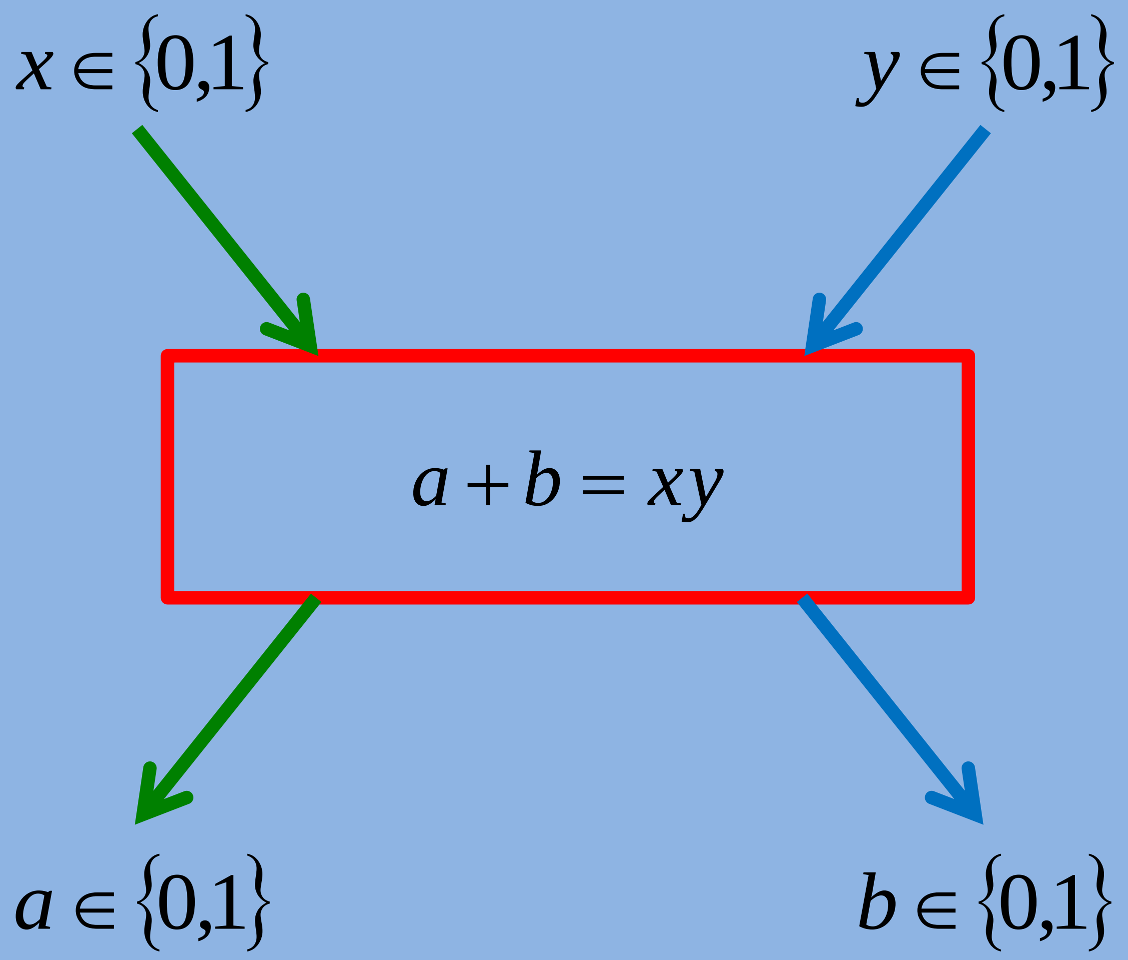
Sandu Popescu

Daniel Rohrlich

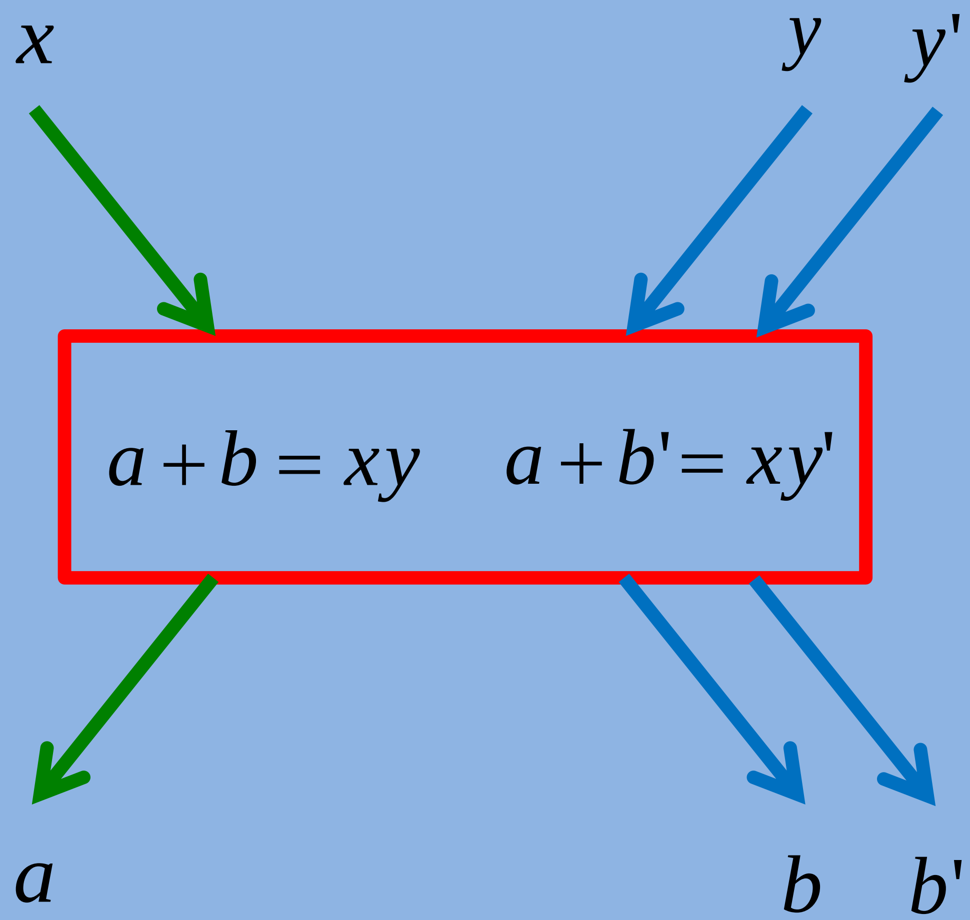
		$P_{AB}(?,1)$		$P_{AB}(?,-1)$		$P_{AB'}(?,1)$		$P_{AB'}(?,-1)$
		=		=		=		=
$P_{AB}(1,?)$	=	$P_{AB}(1,1)$	+	$P_{AB}(1,-1)$	=	$P_{AB'}(1,1)$	+	$P_{AB'}(1,-1)$
		+	E_{AB}	+		+	$E_{AB'}$	+
$P_{AB}(-1,?)$	=	$P_{AB}(-1,1)$	+	$P_{AB}(-1,-1)$	=	$P_{AB'}(-1,1)$	+	$P_{AB'}(-1,-1)$
		=		=		=		=
$P_{A'B}(1,?)$	=	$P_{A'B}(1,1)$	+	$P_{A'B}(1,-1)$	=	$P_{A'B'}(1,1)$	+	$P_{A'B'}(1,-1)$
		+	$E_{A'B}$	+		+	$E_{A'B'}$	+
$P_{A'B}(-1,?)$	=	$P_{A'B}(-1,1)$	+	$P_{A'B}(-1,-1)$	=	$P_{A'B'}(-1,1)$	+	$P_{A'B'}(-1,-1)$

		$P_{AB}(?,1)$		$P_{AB}(?,-1)$		$P_{AB'}(?,1)$		$P_{AB'}(?,-1)$
		‖		‖		‖		‖
$P_{AB}(1,?)$	=	1/2	+	0	=	1/2	+	0
		+	E_{AB}	+		+	$E_{AB'}$	+
$P_{AB}(-1,?)$	=	0	+	1/2	=	0	+	1/2
		‖		‖		‖		‖
$P_{A'B}(1,?)$	=	1/2	+	0	=	0	+	1/2
		+	$E_{A'B}$	+		+	$E_{A'B'}$	+
$P_{A'B}(-1,?)$	=	0	+	1/2	=	1/2	+	0

Popescu-Rohrlich Box



no-cloning



$$b + b' = x(y + y')$$

Gisin and Massar

$$1 \to M$$

$$F = \frac{2M+1}{3M}$$

pseudospin representation

$$\left| \hat{n}\ ; j = \frac{M}{2}, m \right\rangle$$

$$= \frac{P\left\{ \underbrace{\left|\hat{n}\right\rangle \otimes \ldots \otimes \left|\hat{n}\right\rangle}_{j+m} \otimes \underbrace{\left|-\hat{n}\right\rangle \otimes \ldots \otimes \left|-\hat{n}\right\rangle}_{j-m} \right\}}{\sqrt{\begin{pmatrix} 2j \\ j+m \end{pmatrix}}}$$

cloning machine states

$$\left\langle R_{jm}\left(\hat{n}\right)\middle| R_{jm'}\left(\hat{n}\right)\right\rangle = \delta_{mm'}$$

$$m, m' = -j,\ -j+1, \ldots,\ \ j-1,\ \ j$$

cloning transformation

$$|\hat{n}\rangle \otimes \underbrace{|0\rangle \otimes \ldots \otimes |0\rangle}_{M-1} \otimes |R\rangle$$

$$\downarrow$$

$$\sum_{m=-j}^{j} a_{jm} |\hat{n}\ ; jm\rangle \otimes |R_{jm}(\hat{n})\rangle$$

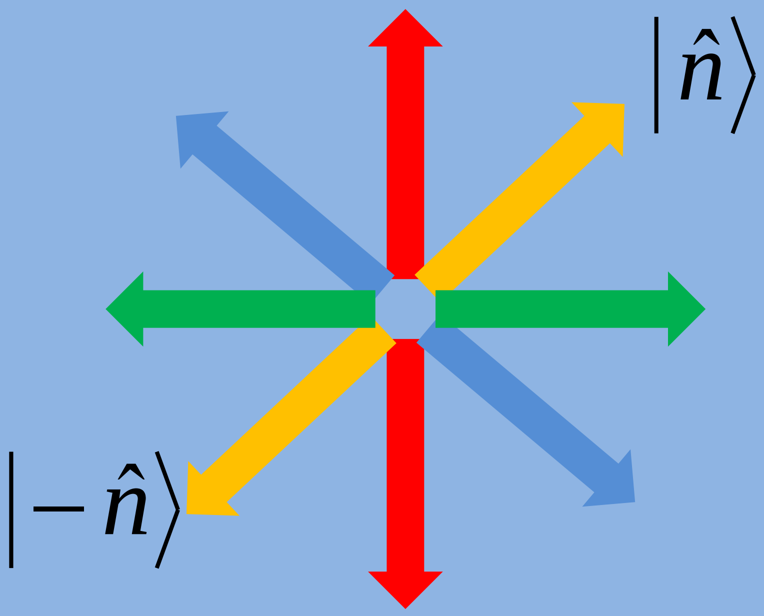
$$\sum_{m=-j}^{j} p_{jm} = 1, \quad p_{jm} = |a_{jm}|^2$$

reduced transformation and fidelity

$$T_j\left(\,|\hat{n}\rangle\langle\hat{n}|\, \right) = \sum_{m=-j}^{j} p_{jm} |\hat{n}\;;jm\rangle\langle\hat{n}\;;jm|$$

$$F_j = \frac{1}{2}\left(1 + \frac{1}{j}\sum_{m=-j}^{j} m p_{jm}\right)$$

no signaling



indistinguishable mixtures

$$\frac{1}{2}|\hat{n}\rangle\langle\hat{n}|+\frac{1}{2}|-\hat{n}\rangle\langle-\hat{n}|=I$$

for no-signaling

$$T_j\left(\,|\hat{n}\rangle\langle\hat{n}|\,\right)+T_j\left(\,|-\hat{n}\rangle\langle-\hat{n}|\,\right)$$

$$=\sum_{m=-j}^{j}\left(\,p_{jm}+p_{j,-m}\right)\left|\hat{n}\;;jm\right\rangle\left\langle\hat{n}\;;jm\right|$$

must be rotationally invariant.

$$p_{jm} + p_{j,-m} = \frac{2}{2j+1}$$

$$F_j^{\mathrm{NS}} = \frac{1}{2} - \frac{1}{j(2j+1)} \sum_{m>0}^{j} m + \frac{1}{j} \sum_{m>0}^{j} m p_{jm}$$

$$0 \le p_{jm} \le \frac{2}{2j+1}$$

no-signaling…

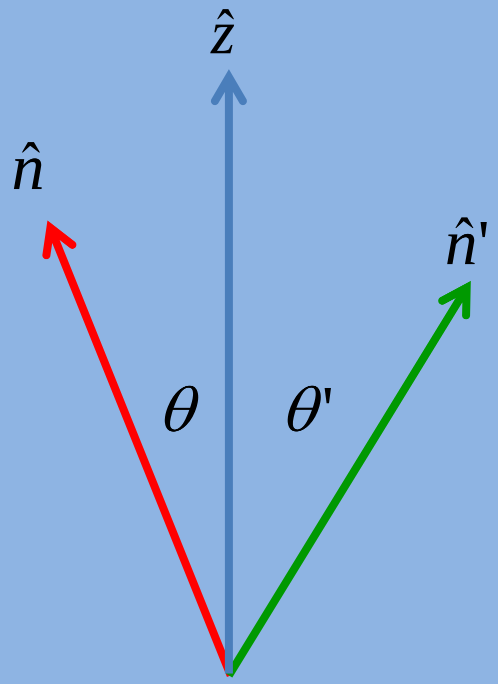
$$r|\hat{n}\rangle\langle\hat{n}| + (1-r)\,|\hat{n}'\rangle\langle\hat{n}'| = s|\hat{m}\rangle\langle\hat{m}| + (1-s)\,|-\hat{m}\rangle\langle-\hat{m}|$$

$$\Downarrow$$

$$T_j\left(\ r|\hat{n}\rangle\langle\hat{n}| + (1-r)\,|\hat{n}'\rangle\langle\hat{n}'|\ \right) = T_j\left(\ s|\hat{m}\rangle\langle\hat{m}| + (1-s)\,|-\hat{m}\rangle\langle-\hat{m}|\ \right)$$

$$0 \le r, s \le 1$$

eigenvalue problem



$$\sum_{m'=-j}^{j}\left(\left|d^{(j)}_{mm'}(\theta)\right|^2 \sin\theta' + \left|d^{(j)}_{mm'}(\theta')\right|^2 \sin\theta + \frac{\sin(\theta+\theta') - \sin\theta - \sin\theta'}{2j+1}\right) p_{jm'}$$

$$= \sin(\theta+\theta')\, p_{jm}$$

$$\therefore p_{jm}(t) = \frac{1}{2j+1} + \frac{2t-1}{j(2j+1)} m$$

$$0 \le t \le 1$$

$$F_j^{\mathrm{Q}}(t) = \frac{1}{6j}\left[(2j-1) + 2(j+1)\ t\right]$$

$$\Downarrow$$

$$\left(F_j^{\mathrm{Q}}\right)_{\max} = \frac{4j+1}{6j}$$

Optimal Quantum Cloning Machines

N. Gisin[1] and S. Massar[2]
[1]Group of Applied Physics, University of Geneva, 1211 Geneva, Switzerland
[2]Raymond and Beverly Sackler Faculty of Exact Sciences, School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69978, Israel
(Received 27 May 1997)

We present quantum cloning machines that transform N identical qubits into $M > N$ identical copies and we prove that the fidelity (quality) of these copies is optimal. The connection between cloning and measurement is discussed in detail. When the number of clones M tends towards infinity, the fidelity of each clone tends towards the optimal fidelity that can be obtained by a measurement on the input qubits. More generally, quantum cloning machines are universal devices to translate quantum information into classical information. [S0031-9007(97)03916-1]

A somewhat lengthy computation involving combinatorial series shows that this unitary operator acts on an arbitrary input state ψ as follows:

$$U_{1,M}|\psi\rangle \otimes R = \sum_{j=0}^{M-1} \alpha_j |(M-j)\psi, j\psi^{\perp}\rangle \otimes R_j(\psi), \quad (3)$$

fidelity

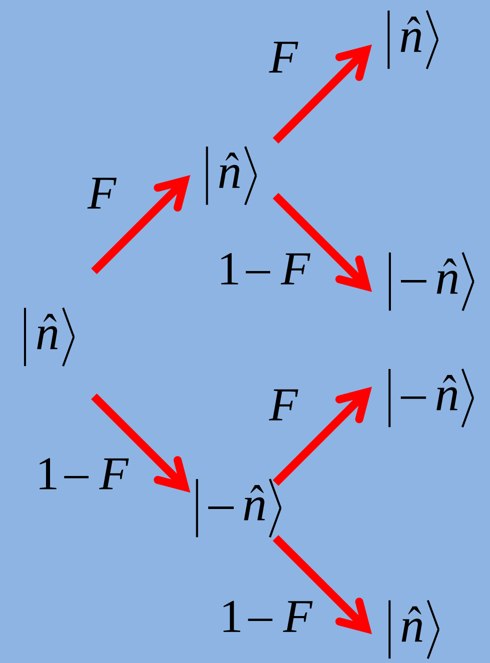
↓

unique quantum cloner

“prime” cloning

$1 \rightarrow 2$ → $1 \rightarrow 3$

$1 \rightarrow 6$



$$F^{\mathrm{P}}_{M/2} F^{\mathrm{P}}_{N/2} + \left(1 - F^{\mathrm{P}}_{M/2}\right)\left(1 - F^{\mathrm{P}}_{N/2}\right) = F^{\mathrm{P}}_{MN/2}$$

$$p_{jm} = \frac{1}{2j+1}\left(1 + \frac{3m}{2j(j+1)}\right)$$

$$F_j^{\mathrm{P}} = \frac{2j+1}{4j}$$

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