

# **Local Realism, Non-Contextuality and Cloning**

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Sabancı University

# Bell-Clauser-Horne-Shimony-Holt

$a$

$b$

$\pm 1$

$b'$

$a'$

# Bell-Clauser-Horne-Shimony-Holt

$$S \equiv ab + ab' + a'b - a'b'$$

$$= (a + a')b + (a - a')b'$$

$$a = a'$$



$$2ab$$

$$a \neq a'$$



$$2ab'$$

$$|S| \leq 2$$

# Bell-Clauser-Horne-Shimony-Holt Quantum

$$S = \langle AB \rangle + \langle AB' \rangle + \langle A' B \rangle - \langle A' B' \rangle$$

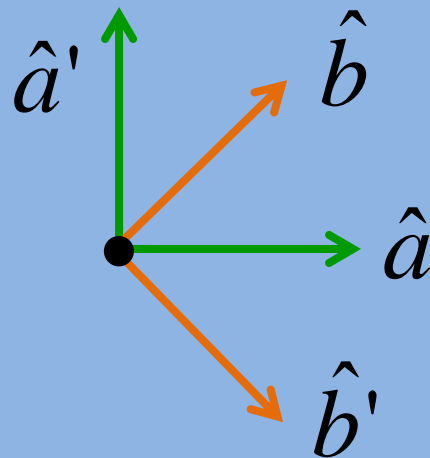


$$2\sqrt{2}$$

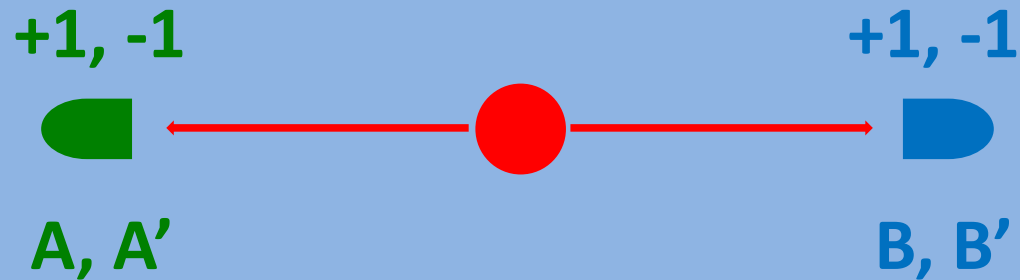
# Bell-Clauser-Horne-Shimony-Holt Quantum

$$|\Psi\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

$$A = \vec{\sigma} \cdot \hat{a}, \quad A' = \vec{\sigma} \cdot \hat{a}', \quad B = \vec{\sigma} \cdot \hat{b}, \quad B' = \vec{\sigma} \cdot \hat{b}'$$



# Bell-Clauser-Horne-Shimony-Holt



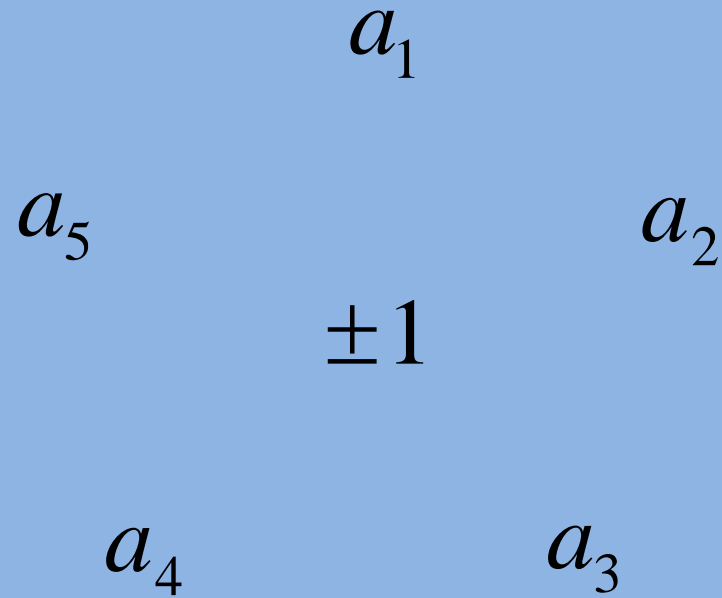
$$E_{AB} = P_{AB}(=) - P_{AB}(\neq) = \langle AB \rangle$$

$$S = E_{AB} + E_{AB'} + E_{A'B} - E_{A'B'}$$

$$|S| \leq 2$$

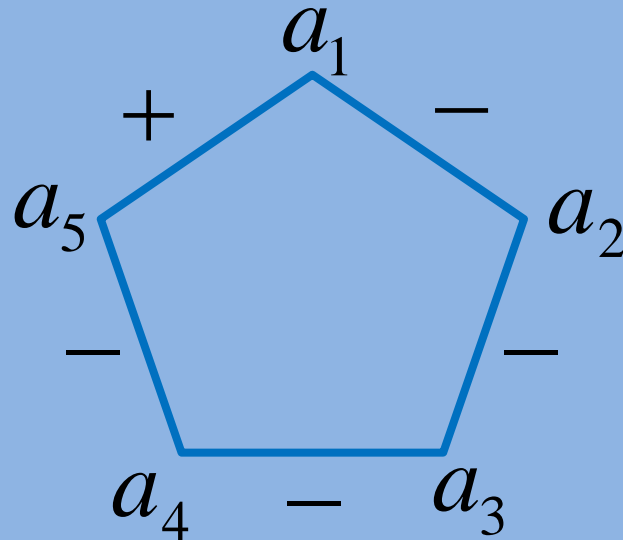
**Local Realism**

# Klyachko-Can-Binicioğlu-Shumovsky



# Klyachko-Can-Binicioğlu-Shumovsky

$$S \equiv a_1 a_2 + a_2 a_3 + a_3 a_4 + a_4 a_5 + a_5 a_1$$



$$S \geq -3$$



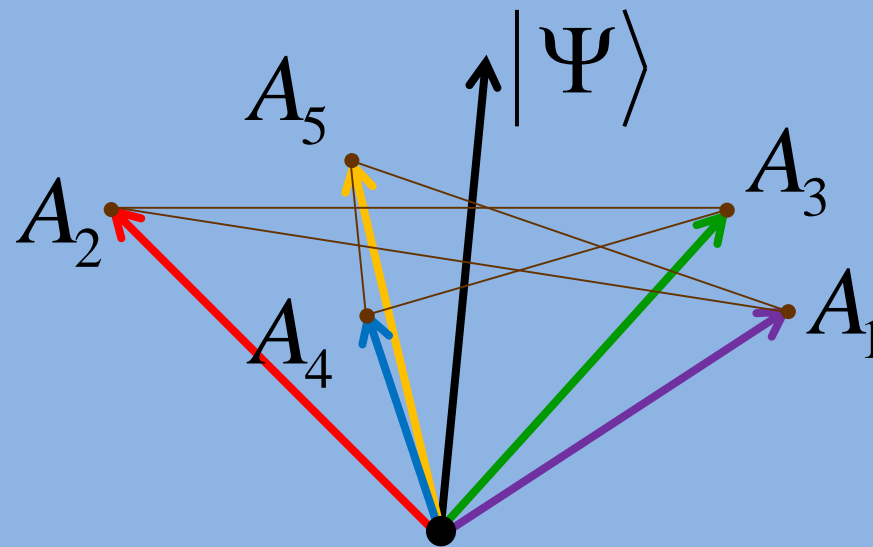
# Klyachko-Can-Binicioğlu-Shumovsky Quantum

$$S = \langle A_1 A_2 \rangle + \langle A_2 A_3 \rangle + \langle A_3 A_4 \rangle + \langle A_4 A_5 \rangle + \langle A_5 A_1 \rangle$$



$$5 - 4\sqrt{5}$$

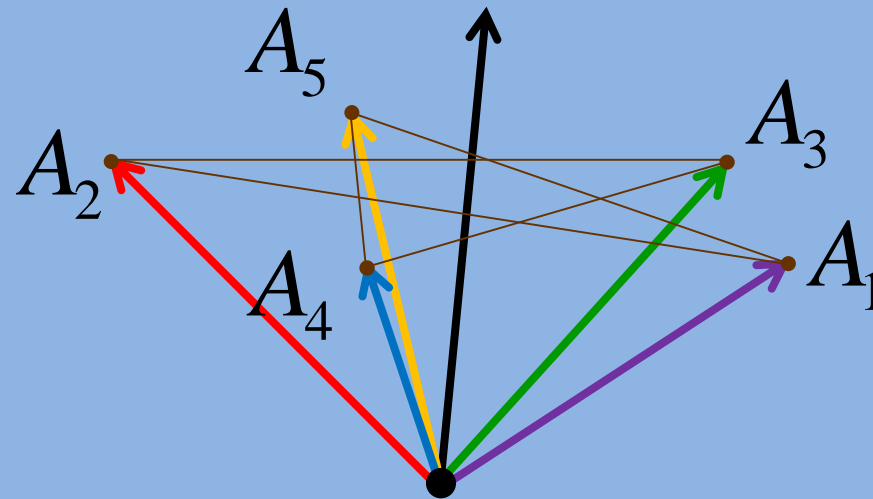
# Klyachko-Can-Binicioğlu-Shumovsky Quantum



$$A_i = 2S_i^2 - I$$

spin-1

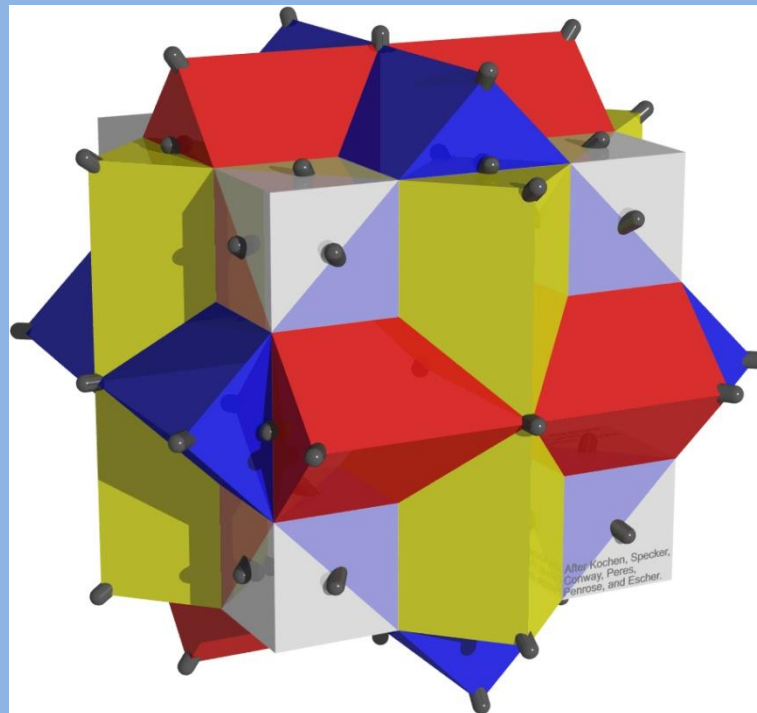
# Klyachko-Can-Binicioğlu-Shumovsky



**Non-Contextuality**

# Kochen-Specker Theorem

Quantum mechanics is contextual.  
Results of quantum mechanics cannot be fully explained  
by non-contextual theories which assume that  
the measurement outcomes of a physical system are  
*predetermined* and independent of their own  
and other simultaneous compatible measurements.



# Monogamy of CHSH and KCBS Inequalities

$$\left| S_{CHSH}^{AB} \right| + \left| S_{CHSH}^{AC} \right| \leq 4$$

$$S_{CHSH}^{AB} + S_{KCBS}^A \geq -5$$





**Filiz**

**FE FE FE FE**

$$\frac{|00\rangle\langle 00| + |11\rangle\langle 11|}{2} = \frac{|++\rangle\langle ++| + |--\rangle\langle --|}{2}$$

**E E E**

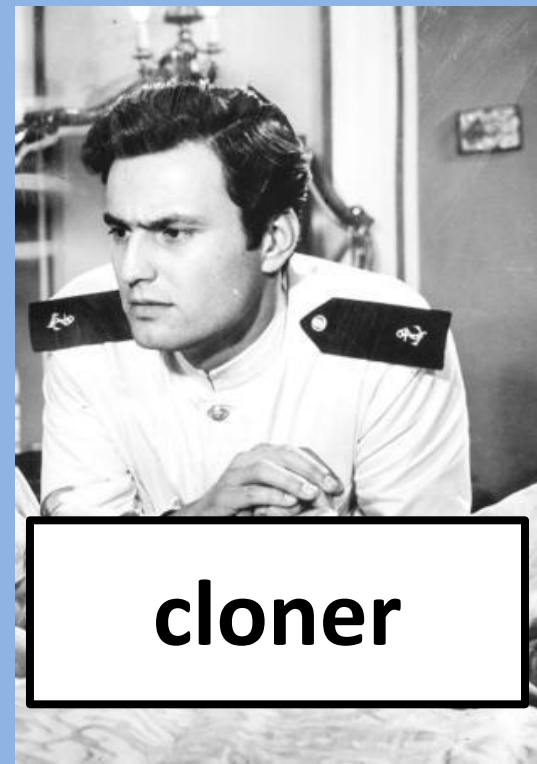
$$|0\rangle \rightarrow |0\rangle \otimes \dots \otimes |0\rangle$$

$$|1\rangle \rightarrow |1\rangle \otimes \dots \otimes |1\rangle$$

$$|+\rangle \rightarrow |+\rangle \otimes \dots \otimes |+\rangle$$

$$|-\rangle \rightarrow |-\rangle \otimes \dots \otimes |-\rangle$$

**Ediz**



**cloner**

# ANNALEN DER PHYSIK.

BRUNNEN- UND POLYTECHNISCHE VERLAGS-ANSTALT

F. A. C. GREIN, L. W. GILBERT, J. C. POGGENDORFF,

VIERTE FOLGE.

BAND 17.

DES ANNIES 1905.

KURATIR

F. KOHLRAUSCH, M. PLANCK, G. QUINCEY,

W. C. RÖNTGEN, P. WARBURG.

UNTER MITWIRKUNG

DER DEUTSCHEN PHYSIKALISCHEN GESELLSCHAFT

HERAUSGEBEN VON

PAUL DRUDE

MIT FÜNF FIGURENTAFELN



LEIPZIG, 1905.

MANN AMBROSIIUS BARTH.

## 3. Zur Elektrodynamik bewegter Körper; von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhafsten scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Magneten und einem Leiter. Das beobachtbare Phänomen hängt von der relativen Bewegung des Leiters und Magneten ab. In beiden Fällen, ob der Magnet sich bewegt, während der Leiter ruht, oder der andere dieser Körper der bewegte sei, streng voneinander zu trennen sind. Bewegt sich nämlich der Magnet, so entsteht in der Umgebung des Magneten ein elektrisches Feld von gewissem Energiewerte, welches an Orten, wo sich Teile des Leiters befinden, einen Strom induziert. Richt aber der Magnet ruht, während sich der Leiter bewegt, so induziert der Leiter ein elektrisches Feld, welches im Leiter eine elektromotorische Kraft erzeugt, welche keine Energie entspricht, die aber — Gleichung (1) — der Bewegung bei der Bewegung des Leiters entspricht. In beiden Fällen selbst aber die gleiche Erscheinung von derselben Art, welche an demselben Verlaufe Veranlassung gibt, wie im ersten Falle, ist die induzierten elektrischen Kräfte.

Die Erscheinung, daß die Erde relativ zum „Lichtmedium“ zu konstanter Geschwindigkeit sich bewegt, führt zu der Vermutung, daß dem Begriffe der Ruhe nicht nur in der Mechanik, sondern auch in der Elektrodynamik eine Bedeutung beizulegen ist. Es ist aber nicht das, was man gemeinlich die Erscheinungen entzerrt, sondern daß vielmehr für alle Koordinatensysteme, die mechanischen Gleichungen gelten, auch die elektrodynamischen und optischen Gesetze gelten, wie in der ersten Ordnung bereits erwiesen ist. Wir vermuten (deren Inhalt im folgenden „Prinzip“ genannt werden wird) zur Voraussetzung erforderlich, daß die mit ihm nur scheinbar unverträgliche

**special relativity (1905)**  
**The speed of light in vacuum is the maximum speed at which all energy, matter, and information in the universe can travel.**

**cloning 0**

$$|0\rangle|0\rangle|0\rangle$$



$$\sqrt{\frac{2}{3}}|0\rangle|0\rangle|0\rangle + \sqrt{\frac{1}{6}}(|0\rangle|1\rangle + |1\rangle|0\rangle)|1\rangle$$

**cloning 1**

$$|1\rangle|0\rangle|0\rangle$$



$$\sqrt{\frac{2}{3}}|1\rangle|1\rangle|1\rangle + \sqrt{\frac{1}{6}}(|1\rangle|0\rangle + |0\rangle|1\rangle)|0\rangle$$



$$|\psi\rangle|0\rangle|0\rangle$$



$$\sqrt{\frac{2}{3}}|\psi\rangle|\psi\rangle|\psi^*\rangle + \sqrt{\frac{1}{6}}\left(|\psi\rangle|\psi^\perp\rangle + |\psi^\perp\rangle|\psi\rangle\right)|\psi^{*\perp}\rangle$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi^\perp\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$$

$$|\psi\rangle = \alpha^*|0\rangle + \beta^*|1\rangle$$

# fidelity

$$\rho_A = \frac{5}{6} |\psi\rangle\langle\psi| + \frac{1}{6} |\psi^\perp\rangle\langle\psi^\perp|$$

$$F_A = \langle\psi|\rho_A|\psi\rangle = \frac{5}{6}$$

# Gisin and Massar

$$1 \rightarrow M$$

$$F = \frac{2M + 1}{3M}$$



ELSEVIER

18 May 1998

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PHYSICS LETTERS A

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Physics Letters A 242 (1998) 1-3

# Quantum cloning without signaling

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## Abstract

Perfect quantum cloning machines (QCM) would allow one to use quantum non-locality for arbitrary fast signaling. However, perfect QCM cannot exist. We derive a bound on the fidelity of QCM compatible with the no-signaling constraint. This bound equals the fidelity of the Bužek–Hillery QCM. © 1998 Elsevier Science B.V.

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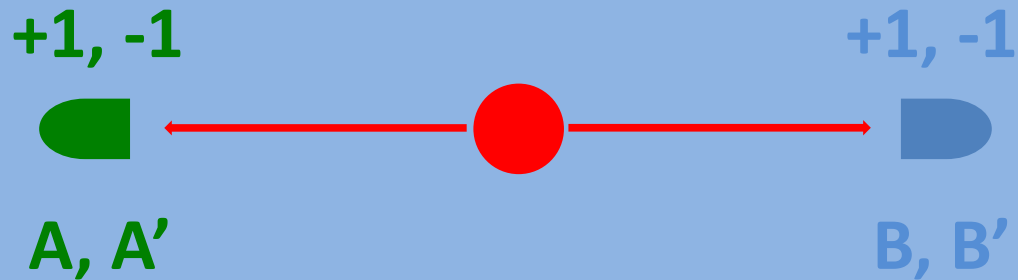


**Nicolas Gisin**

$$1 \rightarrow 2$$

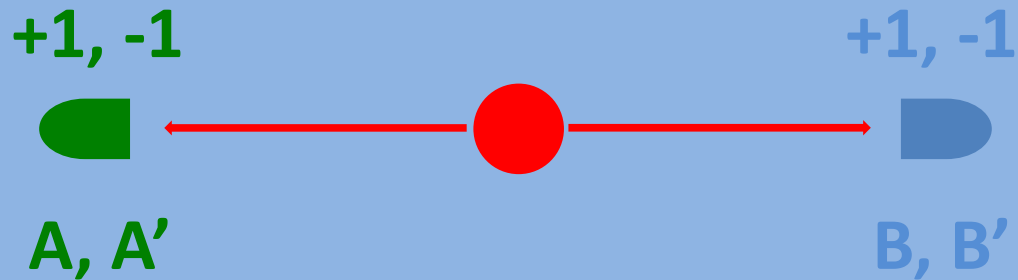
$$\left(F_{M=2}^{NS}\right)_{\max} = \frac{5}{6} = \left(F_{M=2}^Q\right)_{\max}$$

A quite straightforward proof of the optimality of the Bužek–Hillery QCM [3] has been presented, based on the fact that no quantum process can provide arbitrarily fast signaling [7]. Once again, quantum mechanics is **right at the border line of contradicting relativity**, but does not cross it. The peaceful coexistence between quantum mechanics and relativity [1] is thus re-enforced. It is intriguing that the no signaling constraint is a powerful guide to find the limits of quantum mechanics [8].



$$P_{AB}(1,1) + P_{AB}(1,-1) = P_{AB}(1,?) = P_{AB'}(1,1) + P_{AB'}(1,-1)$$

**no-signaling**



$$E_{AB} = P_{AB}(=) - P_{AB}(\neq) = \langle AB \rangle$$

$$S = E_{AB} + E_{AB'} + E_{A'B} - E_{A'B'}$$

$$|S| \leq 2$$

$$2 \leq |S| \leq 2\sqrt{2}$$

**superquantum nonlocal correlations**



**stronger violation of Bell inequalities**



**Sandu Popescu**



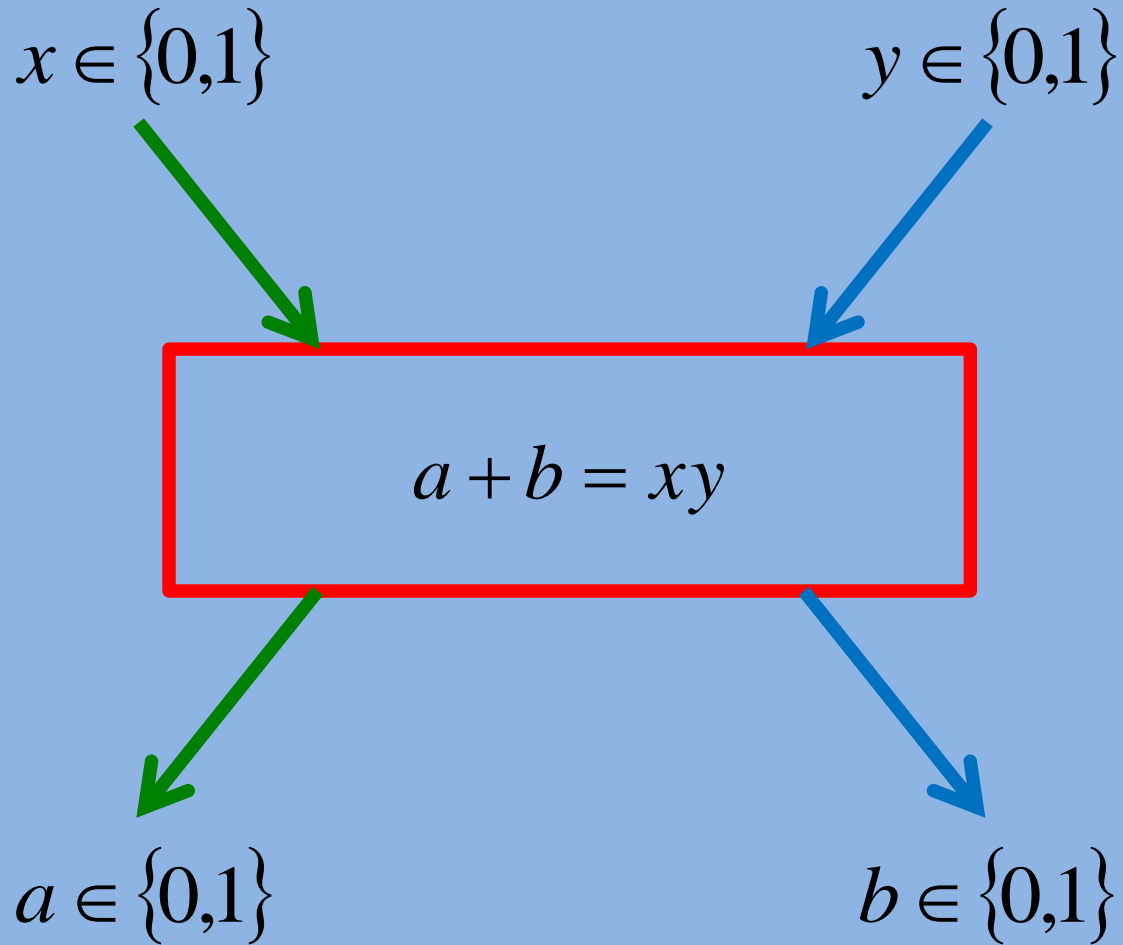
**Daniel Rohrlich**



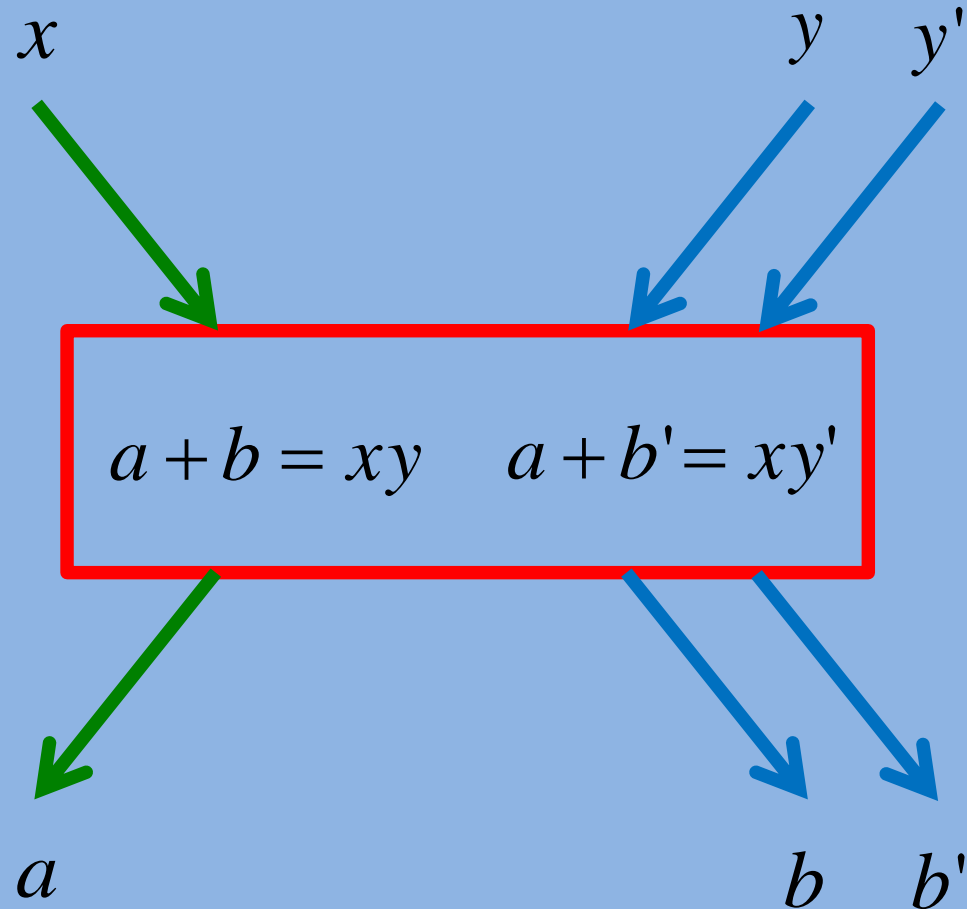
		$P_{AB}(?,1)$		$P_{AB}(?,-1)$		$P_{A'B'}(?,1)$		$P_{A'B'}(?,-1)$
$P_{AB}(1,?)$	=	$P_{AB}(1,1)$	+	$P_{AB}(1,-1)$	=	$P_{A'B'}(1,1)$	+	$P_{A'B'}(1,-1)$
		+	$E_{AB}$	+		+	$E_{A'B'}$	+
$P_{AB}(-1,?)$	=	$P_{AB}(-1,1)$	+	$P_{AB}(-1,-1)$	=	$P_{A'B'}(-1,1)$	+	$P_{A'B'}(-1,-1)$
$P_{A'B'}(1,?)$	=	$P_{A'B'}(1,1)$	+	$P_{A'B'}(1,-1)$	=	$P_{A'B'}(1,1)$	+	$P_{A'B'}(1,-1)$
		+	$E_{A'B}$	+		+	$E_{A'B'}$	+
$P_{A'B'}(-1,?)$	=	$P_{A'B'}(-1,1)$	+	$P_{A'B'}(-1,-1)$	=	$P_{A'B'}(-1,1)$	+	$P_{A'B'}(-1,-1)$

		$P_{AB}(?,1)$		$P_{AB}(?,-1)$		$P_{A'B'}(?,1)$		$P_{A'B'}(?,-1)$
$P_{AB}(1,?)$	=	1/2	+	0	=	1/2	+	0
		+	$E_{AB}$	+		+	$E_{A'B'}$	+
$P_{AB}(-1,?)$	=	0	+	1/2	=	0	+	1/2
$P_{A'B'}(1,?)$	=	1/2	+	0	=	0	+	1/2
		+	$E_{A'B}$	+		+	$E_{A'B'}$	+
$P_{A'B'}(-1,?)$	=	0	+	1/2	=	1/2	+	0

# Popescu-Rohrlich Box



# no-cloning



$$b + b' = x(y + y')$$

# Gisin and Massar

$$1 \rightarrow M$$

$$F = \frac{2M + 1}{3M}$$

# pseudospin representation

$$\left| \hat{n} ; j = \frac{M}{2}, m \right\rangle$$

$$= \frac{P\left\{ \underbrace{|\hat{n}\rangle \otimes \dots \otimes |\hat{n}\rangle}_{j+m} \otimes \underbrace{|-\hat{n}\rangle \otimes \dots \otimes |-\hat{n}\rangle}_{j-m} \right\}}{\sqrt{\binom{2j}{j+m}}}$$

# cloning machine states

$$\langle R_{jm}(\hat{n}) | R_{jm'}(\hat{n}) \rangle = \delta_{mm'}$$

$$m, m' = -j, -j+1, \dots, j-1, j$$

# cloning transformation

$$|\hat{n}\rangle \otimes \underbrace{|\mathbf{0}\rangle \otimes \dots \otimes |\mathbf{0}\rangle}_{M-1} \otimes |R\rangle$$



$$\sum_{m=-j}^j a_{jm} |\hat{n} ; jm\rangle \otimes |R_{jm}(\hat{n})\rangle$$

$$\sum_{m=-j}^j p_{jm} = 1, \quad p_{jm} = |a_{jm}|^2$$

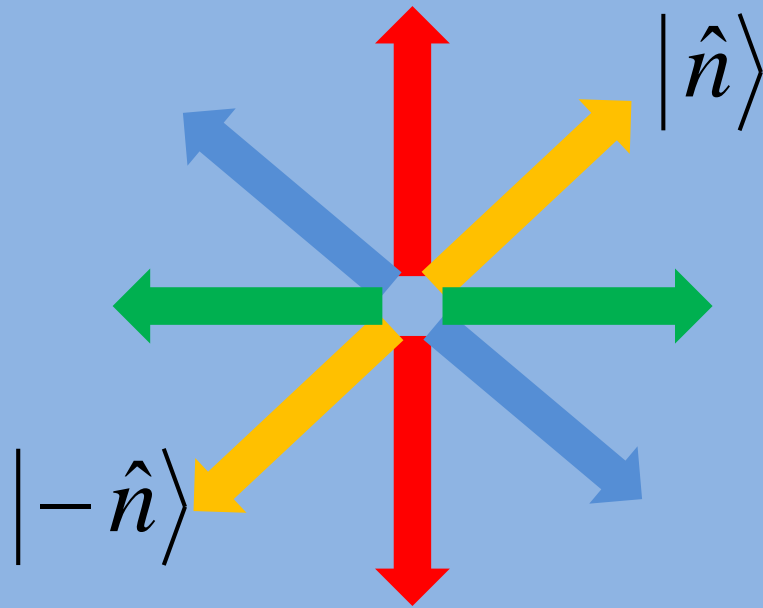


# reduced transformation and fidelity

$$T_j \left( |\hat{n}\rangle\langle\hat{n}| \right) = \sum_{m=-j}^j p_{jm} |\hat{n} ; jm\rangle\langle\hat{n} ; jm|$$

$$F_j = \frac{1}{2} \left( 1 + \frac{1}{j} \sum_{m=-j}^j m p_{jm} \right)$$

**no signaling**



**indistinguishable  
mixtures**

$$\frac{1}{2}|\hat{n}\rangle\langle\hat{n}| + \frac{1}{2}|\hat{-n}\rangle\langle\hat{-n}| = I$$

**for no-signaling**

$$T_j \left( |\hat{n}\rangle\langle\hat{n}| \right) + T_j \left( |-\hat{n}\rangle\langle-\hat{n}| \right) \\ = \sum_{m=-j}^j \left( p_{jm} + p_{j,-m} \right) |\hat{n}; jm\rangle\langle\hat{n}; jm|$$

**must be rotationally invariant.**

$$p_{jm} + p_{j,-m} = \frac{2}{2j+1}$$

$$F_j^{\text{NS}} = \frac{1}{2} - \frac{1}{j(2j+1)} \sum_{m>0}^j m + \frac{1}{j} \sum_{m>0}^j m p_{jm}$$

$$0 \leq p_{jm} \leq \frac{2}{2j+1}$$

# no-signaling...

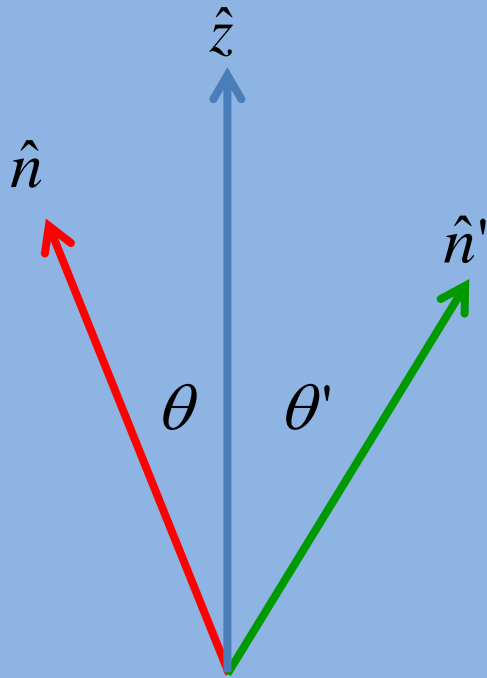
$$r|\hat{n}\rangle\langle\hat{n}| + (1-r)|\hat{n}'\rangle\langle\hat{n}'| = s|\hat{m}\rangle\langle\hat{m}| + (1-s)|-\hat{m}\rangle\langle-\hat{m}|$$

⇓

$$T_j\left(r|\hat{n}\rangle\langle\hat{n}| + (1-r)|\hat{n}'\rangle\langle\hat{n}'|\right) = T_j\left(s|\hat{m}\rangle\langle\hat{m}| + (1-s)|-\hat{m}\rangle\langle-\hat{m}|\right)$$

$$0 \leq r, s \leq 1$$

# eigenvalue problem



$$\sum_{m'=-j}^j \left( \left| d_{mm'}^{(j)}(\theta) \right|^2 \sin \theta' + \left| d_{mm'}^{(j)}(\theta') \right|^2 \sin \theta + \frac{\sin(\theta + \theta') - \sin \theta - \sin \theta'}{2j + 1} \right) P_{jm'}$$
$$= \sin(\theta + \theta') P_{jm}$$

$$\therefore p_{jm}(t) = \frac{1}{2j+1} + \frac{2t-1}{j(2j+1)}m$$

$$0 \leq t \leq 1$$

$$F_j^Q(t) = \frac{1}{6j} [(2j-1) + 2(j+1)t]$$



$$\left(F_j^Q\right)_{\max} = \frac{4j+1}{6j}$$

## Optimal Quantum Cloning Machines

N. Gisin<sup>1</sup> and S. Massar<sup>2</sup>

<sup>1</sup>*Group of Applied Physics, University of Geneva, 1211 Geneva, Switzerland*

<sup>2</sup>*Raymond and Beverly Sackler Faculty of Exact Sciences, School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69978, Israel*

(Received 27 May 1997)

We present quantum cloning machines that transform  $N$  identical qubits into  $M > N$  identical copies and we prove that the fidelity (quality) of these copies is optimal. The connection between cloning and measurement is discussed in detail. When the number of clones  $M$  tends towards infinity, the fidelity of each clone tends towards the optimal fidelity that can be obtained by a measurement on the input qubits. More generally, quantum cloning machines are universal devices to translate quantum information into classical information. [S0031-9007(97)03916-1]

A somewhat lengthy computation involving combinatorial series shows that this unitary operator acts on an arbitrary input state  $\psi$  as follows:

$$U_{1,M}|\psi\rangle \otimes R = \sum_{j=0}^{M-1} \alpha_j |(M-j)\psi, j\psi^\perp\rangle \otimes R_j(\psi), \quad (3)$$

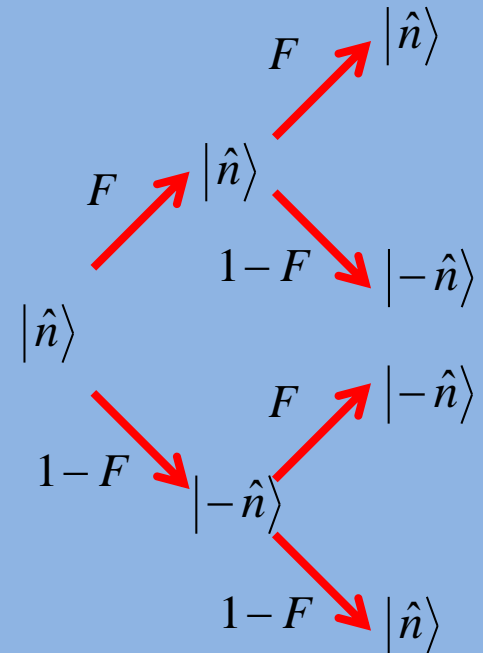
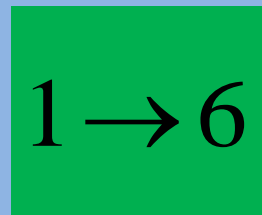
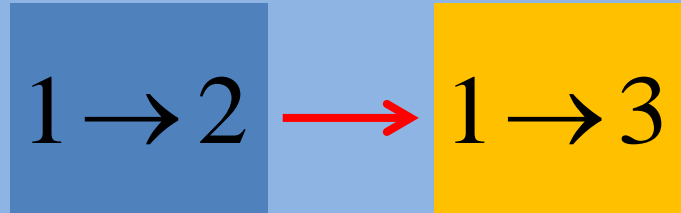


**fidelity**



**unique quantum cloner**

# “prime” cloning



$$F_{M/2}^P F_{N/2}^P + \left(1 - F_{M/2}^P\right) \left(1 - F_{N/2}^P\right) = F_{MN/2}^P$$

$$P_{jm} = \frac{1}{2j+1} \left( 1 + \frac{3m}{2j(j+1)} \right)$$

$$F_j^P = \frac{2j+1}{4j}$$



**111T232**

Z. Gedik and B. Çakmak  
Phys. Rev. A **87**, 042314 (2013)