**HOMEWORK II**
(Due March 06, 2006)

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**Q1:** Let us consider the Dirac equation with a potential \( V(z) \) defined in the half space \( x^3 = z \geq 0 \):

\[
\{i\gamma^\mu \partial_\mu - m - V(z)\} \psi = 0.
\]

The system is invariant under translations in the \( x^1 \) and \( x^2 \) directions as well as rotations around the \( x^3 \) axis. Therefore, without loss of generality, we can make the following ansatz for the solutions:

\[
\psi(x) = e^{-iE_t + ip_1 x^1 + ip_2 x^2} \left( \begin{array}{c} \phi(z) \\ \chi(z) \end{array} \right),
\]

where \( \phi = \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right) \) and \( \chi = \left( \begin{array}{c} \chi_1 \\ \chi_2 \end{array} \right) \) are two-component spinors. Use the standard representation of the Dirac matrices.

a) Find a set of first order differential equations for \( \phi \) and \( \chi \).

b) Derive second-order differential equations for \( \phi \pm i\chi \).

c) Assuming \( m = 0 \), \( p_1 = p_2 = 0 \) and \( V(z) = z \geq 0 \), determine the eigenvalues \( E^2 \).

d) Consider the solutions with the smallest \( E^2 \) in the previous part and \( \sigma_3 = +1 \). Determine the eigenfunctions \( \phi_1 \) and \( \chi_1 \) and the corresponding eigenvalues \( E \).

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**Q2:** For this question, you need to do a little literature search. Give your references properly. I don’t expect you to give a detailed account, but in no less than 1 page for each question, explain

a) What Zitterbewegung is ?

b) What Klein paradox is ?