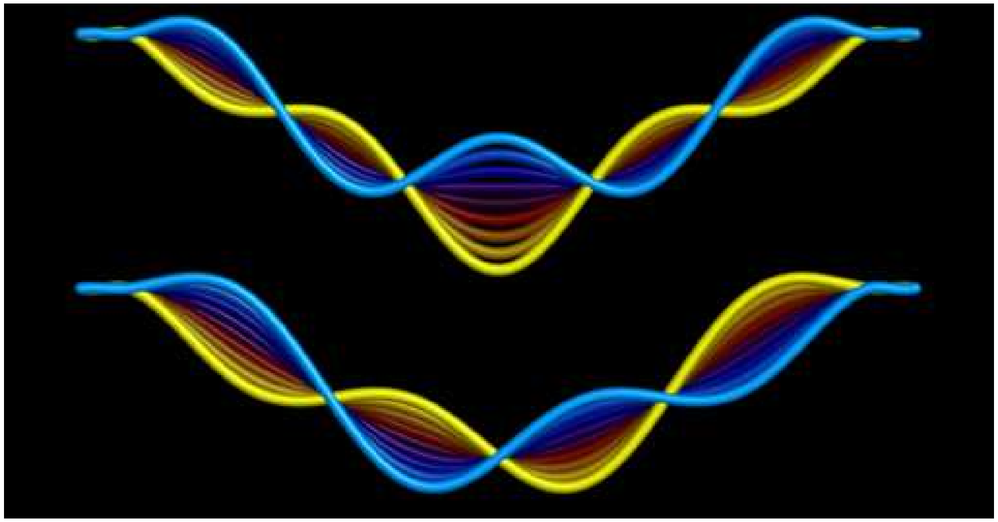
Modelling a Suspended Carbon Nanotube Oscillator



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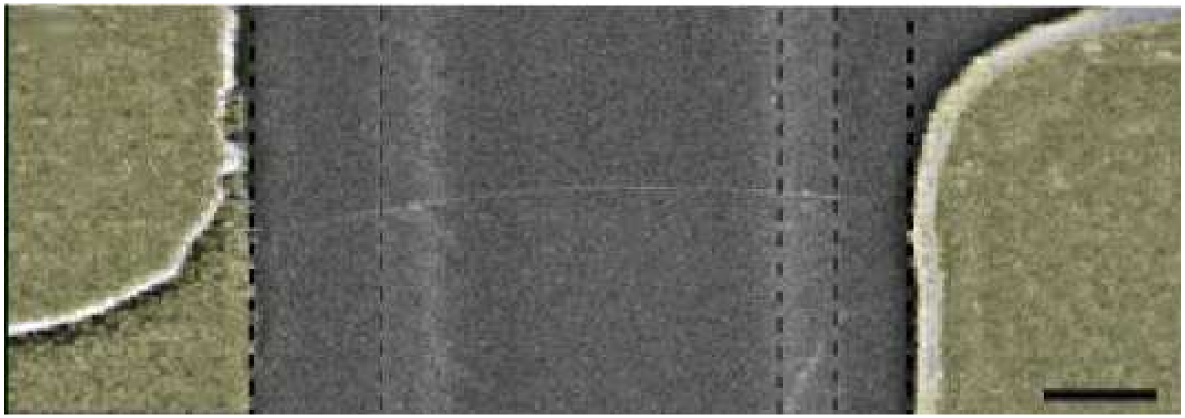
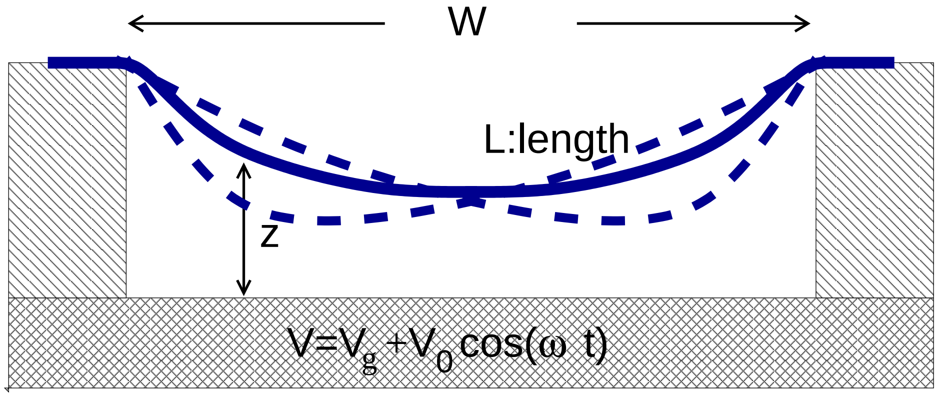
Experiment

Nanotube suspended over trench (courtesy Vera Sazonova)

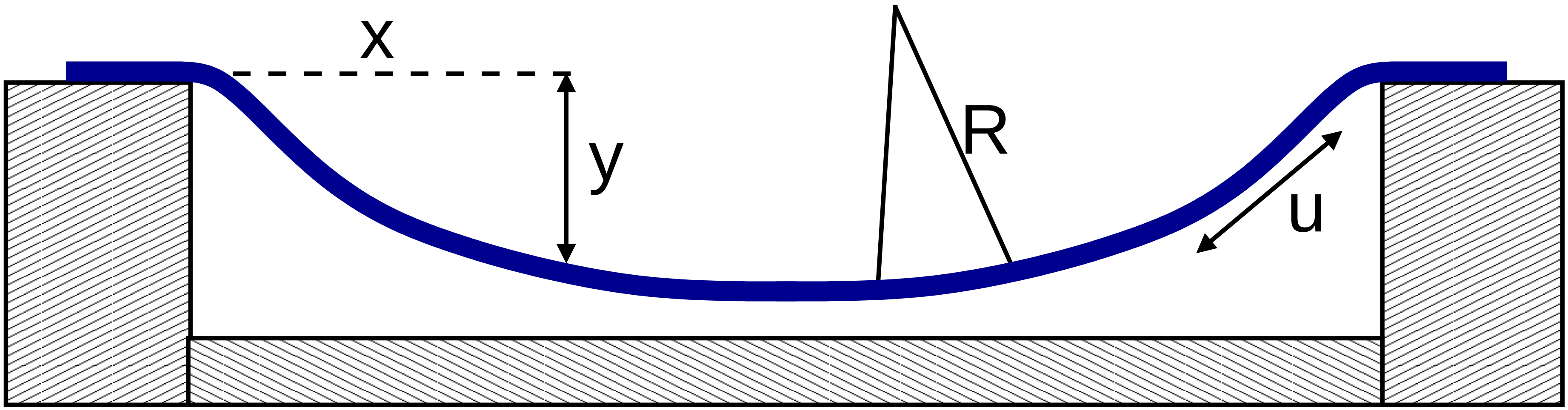




Parameters

Parameter	Typical value
Built-in slack, $s = \frac{L-W}{L}$	0–2%
Gate voltage(DC), V_g	0–6 V

Continuum Model



- Nanotube modeled as an elastic medium with bending and stretching

$$\mathcal{U} = \frac{1}{2}\int_0^L \left[\frac{F}{R^2(x)} + Eu^2(x) + f\,z(x)\right] dx$$

F : bending rigidity
E : extensional rigidity
f : force per length (constant)
$R(x)$: local radius of curvature
$u(x)$: local strain
$z(x)$: downward displacement

Limiting Cases

$$\mathcal{U} = \frac{1}{2}\int_0^L \left[\frac{F}{R^2(x)} + Eu^2(x) + f\,z(x)\right] dx$$

Small V_g

- Bending dominates.
- No extension
- ν is independent of slack, $s = \frac{L-W}{L}$.
- Buckled beam

Intermediate V_g

- No bending
- No extension
- $\nu \propto s^{-1/4}$
- If $s \rightarrow 0$, this limit doesn't exist.
- Hanging chain

Large V_g

- Bending is unimportant.
- Extension dominates.
- ν is independent of s.
- Extended spring

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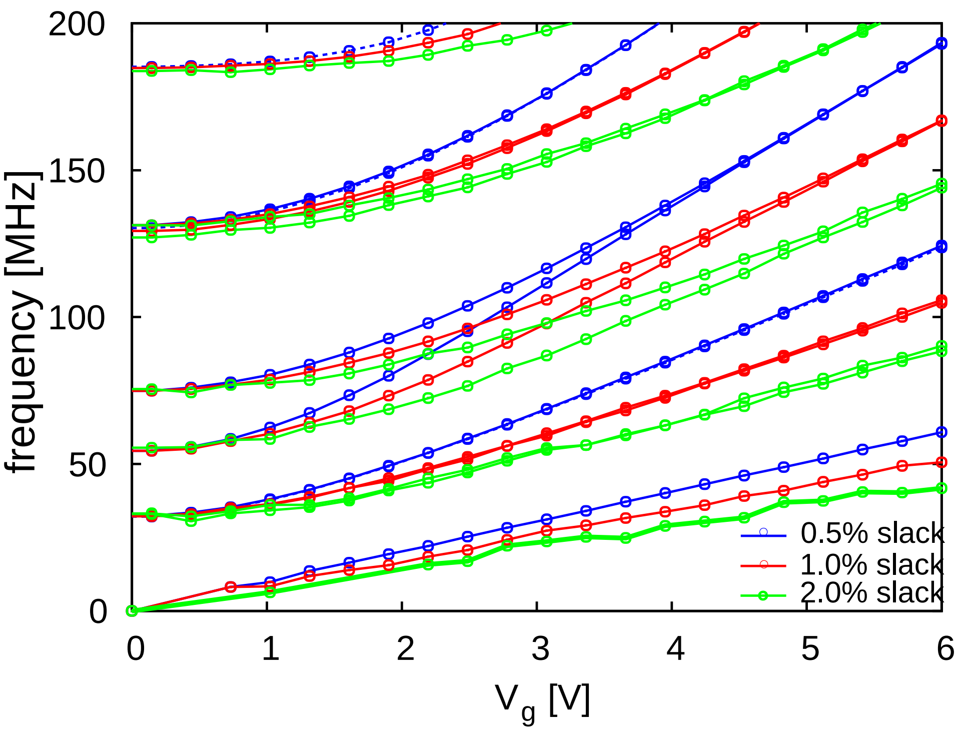
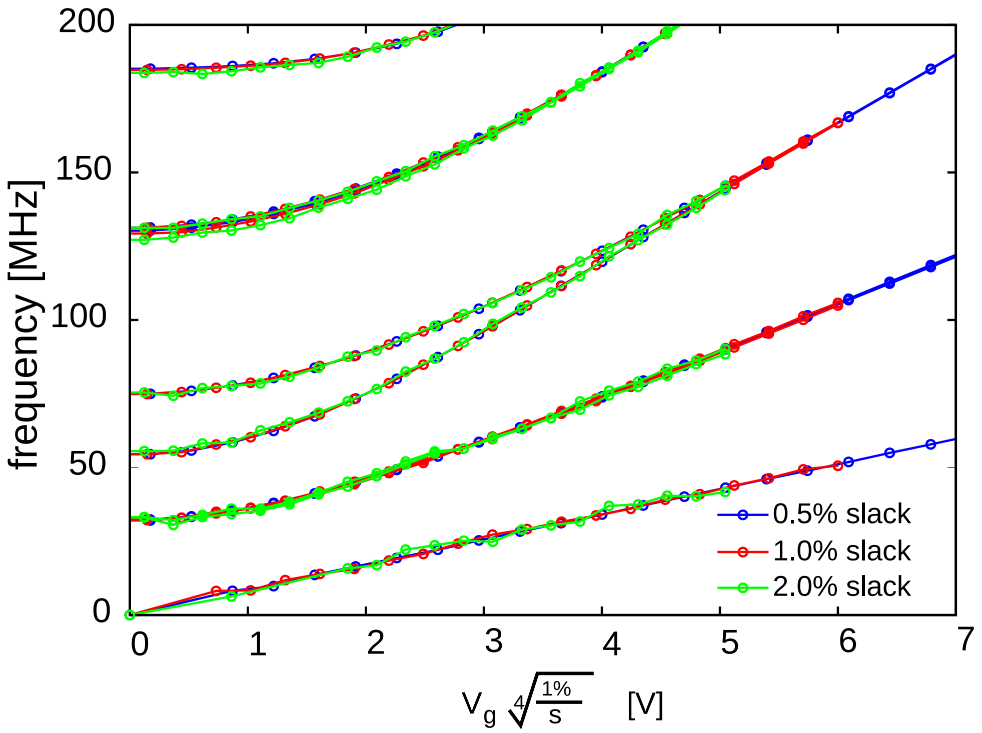
Slack dependence



- Small V_g : ν_n independent of slack
- Large V_g : out of range

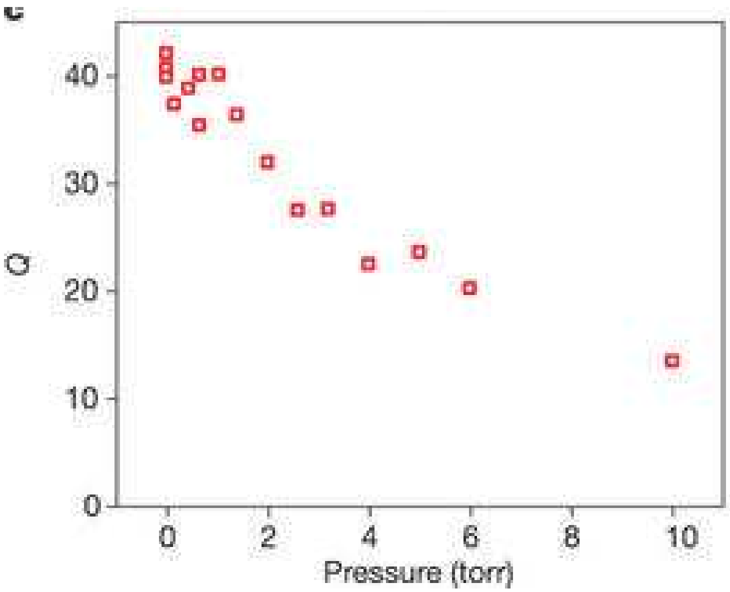


Curves collapse when scaled by $s^{-1/4}$!

- Intermediate $V_g \longrightarrow \nu_n \propto s^{-1/4}$
- Small $V_g \longrightarrow$ get collapsing for free

Phonon–Phonon Losses

Sazonova *et al.* Nature 2004



Nanotubes have low Q-factor

Largest observed ever $\sim$ 1000!

- Phonon–phonon interactions are an intrinsic cause of mechanical loss.
- There are two limits of interest.

Akhieser Limit : $\Omega\tau \ll 1$

- Acoustic phonon disturbs thermal phonon equilibrium.
- Phonons redistribute causing mechanical loss.
- Loss is characterized by inverse quality factor.

$$Q^{-1} \propto \frac{\nu\Omega}{\nu^2+\Omega^2}\int_0^L dx\,\rho^2_{\text{slack}}(x)\,\rho^2_{\text{mode}}(x)\int\frac{dq}{2\pi}\left(\gamma_q n_q^{eq}\right)^2$$

- γ_q describes change in phonon frequencies with radius of curvature.

$$\omega_q = \omega_q^0 + \gamma_q/R^2$$

Ω	: frequency of acoustic mode	ν	: relaxation rate of thermal phonons
$\rho_{\text{slack}}(x)$	: curvature of slack profile	$\rho_{\text{mode}}(x)$	: curvature of acoutic mode
q	: wavevector of thermal phonons	n_q^{eq}	: equilibrium distribution of thermal phonons

Landau-Rümer Limit : $\Omega\tau \gg 1$

- Ballistic collisions between thermal phonons and acoustic phonon cause energy loss.
- Slack and acoustic mode can be treated as athermally populated phonon modes.
- Momentum conversation is satisfied only at band crossings.
- Inverse quality factor :

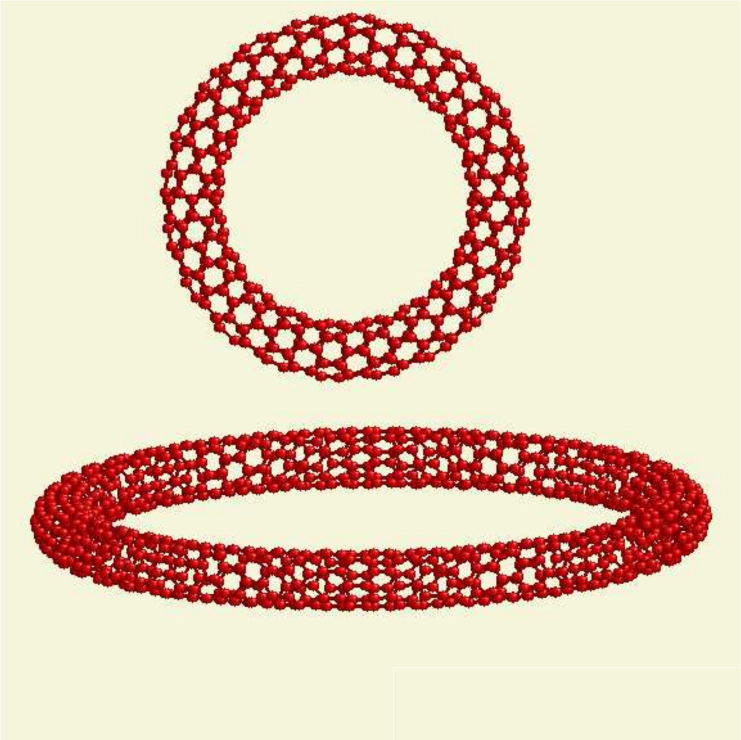
$$Q^{-1} \propto n_q(n_q+1)\frac{1}{\Delta v_g}\frac{1}{\omega_q^2\Omega^2}\underbrace{\left(\frac{\partial\mathcal{U}^4}{\partial^2\rho\,\partial^2\eta_q}\right)^2}_{K}$$

- K causes band gap opening at band crossings.

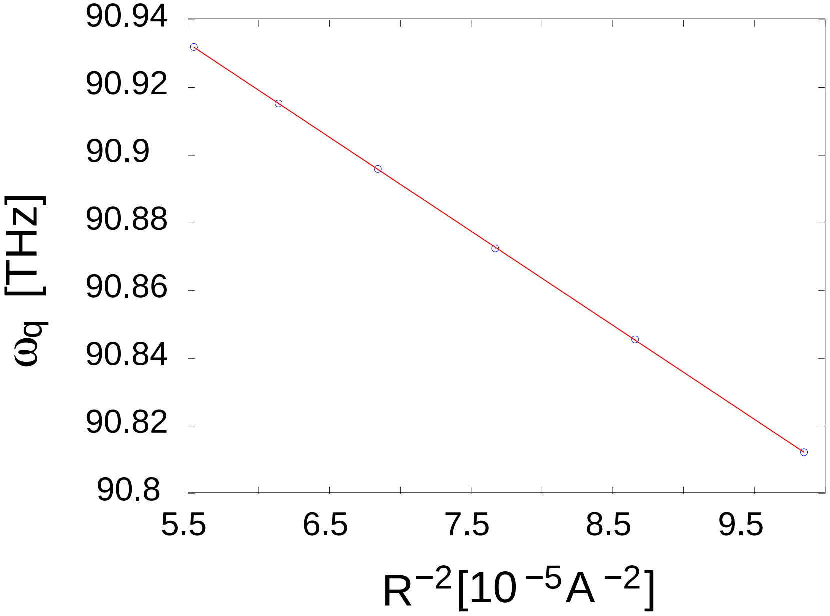
$$\Delta\omega_q = K/m\omega R^2$$

Ω	: frequency of acoustic mode	ω_q	: frequency of thermal phonons
ρ	: radius of curvature	Δv_g	: group velocity difference of thermal phonons
η_q	: amplitude in thermal phonon modes	n_q^{eq}	: equilibrium distribution of thermal phonons

Determination of γ_q and K – Nanotoroids



- γ_q is the slope of ω_q vs R^{-2} plot.



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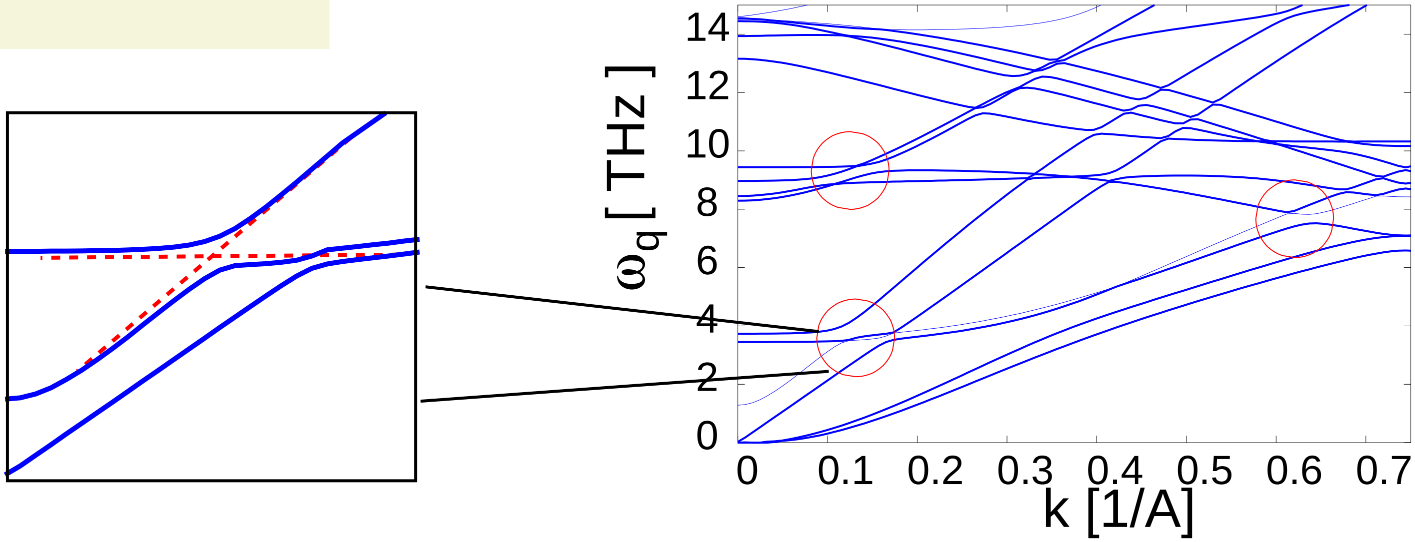

Results and Conclusion

Regime	Quality factor (room temperature)	Quality factor (4 K)
Akhieser	$\approx 10^{14}$	$\approx 10^{63}$
Landau-Rümer	$\approx 10^{5}$	$\approx 10^{23}$

- We have used a continuum model to study suspended nanotubes.
- We have studied mechanical loss due to phonon-phonon coupling.
- Our results indicate that this intrinsic loss mechanism can be ruled out, especially at low temperatures.
- Other possible sources of loss include
  - Clamping
  - Interaction with gas molecules
  - Residual fabrication material left on the nanotube
  - Electron-phonon interaction