Modelling a Suspended Carbon Nanotube Oscillator

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Experiment Los Angeles, 2005

Nanotube suspended over trench (courtesy Vera Sazonova)

\[ V = V_g + V_0 \cos(\omega t) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Built-in slack, ( s = \frac{L-W}{L} )</td>
<td>0–2%</td>
</tr>
<tr>
<td>Gate voltage (DC), ( V_g )</td>
<td>0–6 V</td>
</tr>
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</table>
- Nanotube modeled as an elastic medium with bending and stretching

\[ U = \frac{1}{2} \int_0^L \left[ \frac{F}{R^2(x)} + Eu^2(x) + fz(x) \right] dx \]

- \( F \): bending rigidity
- \( R(x) \): local radius of curvature
- \( E \): extensional rigidity
- \( u(x) \): local strain
- \( f \): force per length (constant)
- \( z(x) \): downward displacement
Limiting Cases

\[
\mathcal{U} = \frac{1}{2} \int_0^L \left[ \frac{F}{R^2(x)} + Eu^2(x) + f z(x) \right] dx
\]

Small \( V_g \)
- Bending dominates.
- No extension
- \( \nu \) is independent of slack, \( s = \frac{L-W}{L} \).
- Buckled beam

Intermediate \( V_g \)
- No bending
- No extension
- \( \nu \propto s^{-1/4} \)
- If \( s \to 0 \), this limit doesn’t exist.
- Hanging chain

Large \( V_g \)
- Bending is unimportant.
- Extension dominates.
- \( \nu \) is independent of \( s \).
- Extended spring
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**Slack dependence**

- Small $V_g$ : $\nu_n$ independent of slack
- Large $V_g$ : out of range

Curves collapse when scaled by $s^{-1/4}$

- Intermediate $V_g \rightarrow \nu_n \propto s^{-1/4}$
- Small $V_g \rightarrow$ get collapsing for free
Sazonova et al. Nature 2004

- Phonon–phonon interactions are an intrinsic cause of mechanical loss.
- There are two limits of interest.

Nanotubes have low $Q$-factor

Largest observed ever $\sim 1000!$
• Acoustic phonon disturbs thermal phonon equilibrium.

• Phonons redistribute causing mechanical loss.

• Loss is characterized by inverse quality factor.

\[
Q^{-1} \propto \frac{\nu \Omega}{\nu^2 + \Omega^2} \int_0^L dx \rho_{\text{slack}}^2(x) \rho_{\text{mode}}^2(x) \int \frac{dq}{2\pi} (\gamma_q n_q^{eq})^2
\]

• \( \gamma_q \) describes change in phonon frequencies with radius of curvature.

\[
\omega_q = \omega_q^0 + \gamma_q / R^2
\]

\( \Omega \) : frequency of acoustic mode
\( \nu \) : relaxation rate of thermal phonons
\( \rho_{\text{slack}}(x) \) : curvature of slack profile
\( \rho_{\text{mode}}(x) \) : curvature of acoustic mode
\( q \) : wavevector of thermal phonons
\( n_q^{eq} \) : equilibrium distribution of thermal phonons
• Ballistic collisions between thermal phonons and acoustic phonon cause energy loss.

• Slack and acoustic mode can be treated as athermally populated phonon modes.

• Momentum conversation is satisfied only at band crossings.

• Inverse quality factor :

\[ Q^{-1} \propto n_q (n_q + 1) \frac{1}{\Delta v_g} \frac{1}{\omega_q^2 \Omega^2} \left( \frac{\partial U^4}{\partial^2 \rho \partial^2 \eta_q} \right)^2 \]

• \( K \) causes band gap opening at band crossings.

\[ \Delta \omega_q = \frac{K}{m \omega R^2} \]

\( \Omega \) : frequency of acoustic mode  \( \omega_q \) : frequency of thermal phonons

\( \rho \) : radius of curvature  \( \Delta v_g \) : group velocity difference of thermal phonons

\( \eta_q \) : amplitude in thermal phonon modes  \( n_q^{eq} \) : equilibrium distribution of thermal phonons
• $\gamma_q$ is the slope of $\omega_q$ vs $R^{-2}$ plot.

• $K$ causes band gap opening.
Results and Conclusion
Los Angeles, 2005

<table>
<thead>
<tr>
<th>Regime</th>
<th>Quality factor (room temperature)</th>
<th>Quality factor (4 K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akhieser</td>
<td>$\approx 10^{14}$</td>
<td>$\approx 10^{63}$</td>
</tr>
<tr>
<td>Landau-Rümer</td>
<td>$\approx 10^5$</td>
<td>$\approx 10^{23}$</td>
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- We have used a continuum model to study suspended nanotubes.
- We have studied mechanical loss due to phonon-phonon coupling.
- Our results indicate that this intrinsic loss mechanism can be ruled out, especially at low temperatures.
- Other possible sources of loss include
  - Clamping
  - Interaction with gas molecules
  - Residual fabrication material left on the nanotube
  - Electron-phonon interaction