

# A Crash Course on Quantum Mechanics

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## A Short History

Quantum theory, like the theory of relativity, owes its birth to some problems associated with light. The apparent constancy of the speed of light led Einstein to develop the theory of relativity. The other problem, that the heated bodies emit only a finite amount of power (and not infinite) led to the birth of quantum mechanics. Both theories are more correct descriptions of nature than the classical laws of physics (Newton's laws of motion and gravity and Maxwell's theory of electromagnetism). The classical laws are only approximate forms of these in certain limiting cases.

You know that the theory of relativity (at least the special one) has some sort of an axiomatic character. Starting from a few basic facts that can be justified easily and verified experimentally, you can obtain most of the results of the theory. Unfortunately, there is no such derivation for the case of quantum mechanics. This is the basic reason for why it took so long for the construction of the complete theory (from 1900 to 1926). A lot of people worked on this theory and uncovered certain aspects of it. This is also why we see a lot of Nobel Prizes awarded on quantum theory. You can read a well-written account of the historical development of the concepts from George Gamow's book "Thirty Years that Shook Physics". In here, I will try to summarize some of these developments.

## Blackbody Radiation and the Birth of Quantum Physics

We first start with the problem that led to the idea of quantum. Unfortunately, the problem is complicated and it is not easy to see how the quantum idea solves it. For the sake of completeness, I will try to give a description of it.

The basic idea is that all objects having a non-zero temperature (which means *all* objects) somehow emit light. (Of course, since visible light is limited to only a certain portion of the entire electromagnetic (EM) spectrum, we should really say that "all objects emit EM waves at various frequencies". Here I will use "light" to mean any EM wave, not just visible light.) The reason of this emission is this: If the object has temperature  $T$ , all atoms of the object have an energy whose average is roughly proportional to  $T$ . They are constantly in motion, for example, the atoms are oscillating around their mean positions, etc. Electrical charges also take part in the thermal motion and for that reason EM waves get produced. When part of these waves escape from the object we observe that the object is emitting light.

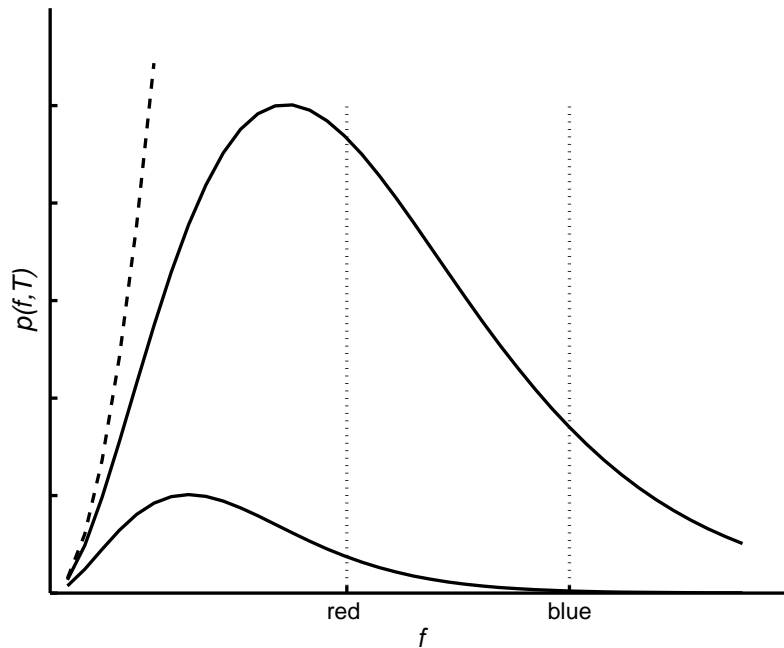


Figure 1: The frequency distribution of blackbody radiation. The upper curve is for Sun ( $T=5800\text{K}$ ) and the lower curve for a cooler body with  $T=3400\text{K}$ . The visible region is indicated (Sun looks white but the cooler object will look red). The classical calculations could only obtain the dashed curve.

Starting with Kirchhoff in 1859, using mainly the theory of thermodynamics, people have derived certain properties of the emitted radiation. It was realized that good absorbers of light are also good emitters (somehow, the laws that govern the absorption of light is closely related to the laws for the emission). For that reason, an object that absorbs all light that falls on it, an ideal blackbody, has the largest emission rate. The EM waves emitted by such a body was therefore called “blackbody radiation”. Experimentally, it was not too difficult to find such bodies which are black at all frequencies, because a hole opened on the walls of an large box with a cavity behaves like an ideal black body at the hole. With this, the blackbody radiation could be studied extensively.

An important property of the blackbody radiation is that it is *universal*. In other words, the nature of the radiation does not depend on the material used. It only depends on the temperature of the object and the frequency of light emitted. For that reason, people tried to determine the properties of this radiation. If it is universal, then it has to be simple. Unfortunately, although some features of it could be derived, most of it remained underivable. Some people started to realize that blackbody radiation contained new physics which cannot be described with the classical laws of physics. For this reason a lot of effort is spent on this problem by a lot of different people.

Most elusive was the frequency distribution of emitted power. Let  $p(f, T)df$  denote the power per unit area of the blackbody which is emitted between frequencies  $f$  and

$f + df$ . Experimental measurements of this quantity indicated that this distribution of power has a bell-shaped curve going to zero at zero and infinite frequencies. However, classical derivations only gave

$$p(f, T) = \frac{2\pi f^2}{c^2} k_B T \quad ,$$

where  $k_B$  is the Boltzmann's constant (gas constant divided by Avagadro's number). Although this expression matches the low frequency part of the distribution, it certainly lead to increasingly large values at high frequencies. The total power emitted

$$P_{tot} = \int_0^{\infty} p(f, T) df$$

should then be infinite. In other words, all objects should lose all their thermal energy instantly and cool to absolute zero of temperature! This problem was later called “ultraviolet catastrophe” as it is that part of the spectrum that causes the problem.

**Max Planck, 1900.** Towards the end of 19th century, a lot of scientist became interested in this problem. Max Planck was one of them trying to obtain the frequency distribution. In one of his derivations, he made a guess on some thermodynamical relationship and came up with the following formula

$$p(f, T) = \frac{2\pi f^2}{c^2} \frac{hf}{\exp(hf/k_B T) - 1} \quad .$$

It was soon realized that this formula fits perfectly to the experimental data. Unsatisfied with his previous derivation, he then went on to find a physical derivation that leads to the same formula. He describes his attempts as an act of desperation: “A theoretical explanation had to be supplied at all costs, whatever the price”. The price was an idea that he didn't like.

First note that, emission of light with frequency  $f$  can only occur if there is an oscillatory motion in the object with frequency  $f$ . So, Planck considered oscillators with a definite frequency. As the blackbody radiation is universal, the exact nature of these oscillators is unimportant. Planck realized that he could obtain his formula if he assumes that an oscillator with frequency  $f$  can only lose an integer multiple of energy  $hf$  to radiation. Here,  $h = 6.6 \times 10^{-34} J \cdot s$  is now called the Planck constant. The energy  $hf$  was called *elementarquantum* (the elementary amount) of energy. Here *quantum* is a Latin word meaning amount (plural is *quanta*), but over time its meaning changed so that it now implies discreteness.

The main problem for Planck was that he didn't believe in “atomic theory”, although the theory was accepted by a lot of scientists at that time. He believed that the matter is continuous down to its smallest details. For this reason, he avoided claiming that light itself is quantized. He also didn't claim that each oscillator has discrete energy values. *Somehow* the energy transfer between oscillators and light was happening in discrete amounts. For this reason, some people call him a revolutionary against his will. He is awarded with the Nobel Prize in 1918 “for his discovery of energy quanta”.

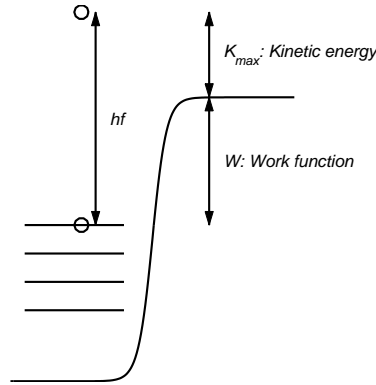


Figure 2: The photoelectric effect. By the absorption of a photon the electron gains an energy  $hf$ . Of this,  $W$  is spent for getting out of the solid. The remaining appears as the kinetic energy of free electrons. Maximum kinetic energy has to be  $K_{max} = hf - W$ .

Let us briefly mention how the idea of quanta explains the blackbody radiation distribution. At a temperature  $T$ , all oscillators of the object has an average energy of the order-of-magnitude of thermal energy,  $k_B T$ . If the energy quantum is much smaller than the thermal energy ( $hf \ll k_B T$ ), then the changes in the oscillator's energy (which are multiples of  $hf$ ) are much smaller than the average energy of the oscillator. For that case, there won't be much difference between continuous energy change assumption and Planck's assumption. In fact Planck's formula reduces to the classical formula in that limit. On the other hand, if the energy quantum is larger than thermal energy ( $hf \gg k_B T$ ), then the oscillator will not have enough energy to emit one quantum. Rarely, the oscillator will have energies much larger than  $k_B T$  and comparable to  $hf$  and only in that case it will be able to emit radiation. The probability of such events to occur is, however, very small, roughly given by the exponential factor  $\exp(-hf/k_B T)$  (a factor that appears in the high frequency limit of Planck's law).

**Albert Einstein, 1905.** Einstein was, of course, a true revolutionary. In his article titled "On a heuristic point of view about the emission and transformation of light" (one of the three articles that we celebrate this year) we see the claim Planck avoided, that the light itself is quantized. For light that occupies a certain space, the part of the total energy that is carried by waves with frequency  $f$  is an integer multiple of  $hf$ . (He reaches to this result by investigating volume dependence of entropy.<sup>1</sup>) He then claims that typically one quantum will enter into processes where light is interacting with matter (emission, absorption or scattering). Armed with these, he proceeds to describe three possible experiments where the quantized nature of light could be observed.

Of these the most important one is the photoelectric effect. It has been observed that when light shines on a metal, electrons are emitted from the surface. In this process, energy from light is absorbed by the electrons. When the electrons have sufficient energy,

<sup>1</sup>An English translation of this article can be obtained from <http://dbserv.ihep.su/hist/owa/hw.part2?s.c=EINSTEIN+1905>

they can get out of the metal. With an appropriate circuit design, the ejected electrons can be observed as a current.

An important quantity in here is the minimum amount of energy needed to extract one electron from the metal,  $W$ , which is traditionally called as the “work function”. It depends on the metal and the surface used and can be measured in various ways. When light with frequency  $f$  is sent to the surface, the electrons can only gain an energy  $hf$  assuming that only one quantum is exchanged in this process. When this energy is smaller than the work function ( $f < W/h$ ), the electron cannot escape the metal and no current should be observed. So, the only effect of light is to heat up the metal in this case.

The fact that below a critical frequency, light cannot cause photoelectric effect cannot be explained with classical electromagnetism. When light is brighter, it carries more energy and therefore it can be expected that the electrons gain more energy. So, the classical theory predicts that the occurrence of photoelectric effect does not depend on the frequency, but it should depend on light intensity. So, what we have here is another phenomenon that cannot be explained by classical laws.

If energy quantum  $hf$  is greater than the work function, then the maximum kinetic energy of the electrons have to be equal to  $K_{max} = hf - W$ . This energy is also dependent only on frequency and not on the intensity of light. Increasing the intensity should only increase the total number of electrons ejected but not their kinetic energy distribution. All of the predictions made by Einstein was verified by a series of experiments by Robert Millikan in 1915. Einstein got the Nobel Prize in 1921 “for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect” and Millikan got the Prize in 1923 “for his work on the elementary charge of electricity and on the photoelectric effect”.

To summarize, what we have in here is a hypothesis that light is made up of discrete units, so called quanta, which will later be called photons. Each photon carries a definite energy  $hf$  directly proportional to its frequency. In all absorption, emission or scattering processes each photon enters into the process in whole, as a complete entity. Each photon also behaves like a particle, it can collide with the other “real” particles like the billiard balls. Experiments in 1923 carried out by Arthur Compton (Nobel Prize in 1927) where photons are scattered off by electrons strengthened this interpretation.

By 1907, Einstein started to apply quantization ideas to another kind of waves, the sound waves in solids. The adaptation should be obvious: A sound wave with frequency  $f$  is formed by particle-like entities (now called phonons) each one having an energy  $hf$ .<sup>2</sup> With this Einstein could calculate the specific heats of solids at all temperatures. He observed that the specific heats go to zero in the limit the temperature  $T$  goes to absolute zero, which had been observed experimentally but otherwise could not be explained at that time.

The ideas above can be extended to all other kinds of waves. This is one important feature of quantum theory that we meet a lot. When we have a continuous medium with the possibility of making some “physical change” in that medium which would “classically” propagate like waves (like the electromagnetic fields in space, or deformations of a solid),

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<sup>2</sup>To be precise, Einstein assumed that all oscillations have same frequency. Later Debye extended this calculation to real sound waves various frequencies.

then the excitations in that medium will appear as particle-like entities. These excitations have fancy names usually ending in “-on”, like graviton for the quanta of gravitational waves or ripplon for the surface waves in liquids. These excitations behave like particles carrying definite energy and momentum and they collide with each other as if they are particles.

This idea can also be extended to the “real” particles, electrons, protons and neutrons (or quarks) etc. The modern view followed in quantum field theory is to consider these particles as excitations of some media (fields). Therefore, the reason matter is formed by discrete units called atoms is exactly this quantization.

**Niels Bohr, 1913.** The other idea that Planck avoided, that the oscillators have quantized energies, is this time taken up by the Danish physicist Niels Bohr. The problem was to understand the existence of atoms. Experiments by Rutherford indicated that the atoms are mostly empty, a heavy positive nucleus is sitting at the center. The only alternative left to the negatively charged electrons was to be in planetary motion around the nucleus.

However, there was an important problem with this picture. An accelerated charge is known to emit EM waves. Since the electrons are in accelerated motion, they should constantly emit these waves and gradually lose energy. We have the same mechanism for the planets orbiting around the Sun as well. However, for the planets the energy lost as gravitational waves is extremely small compared to their total energy. For the case of electrons in an atom, however, the loss is very significant. Calculations show that the electrons, constantly losing energy, should hit the nucleus within a lifetime around  $10^{-8}$  seconds! According to the classical theory, then there should be no atoms.

So, somehow the electrons were not emitting EM waves and Bohr is forced to explain why. Bohr knew that the answer is in quantum theory since an expression he could guess using Planck constant was of the order of the size of atoms. By using certain assumptions he was able to construct a theory which could explain various properties of the Hydrogen atom. Later, together with Arnold Sommerfeld, the theory is extended to include all kinds of physical systems. We now call that theory as the “old quantum theory”. It was only an approximate form of the correct quantum theory, but, surprisingly, it was very successful.

Bohr assumed that the electrons follow the exact orbits which are described by the classical mechanics. However, of these orbits only certain ones are allowed that satisfy the mathematical condition

$$\oint p dx = nh \quad , \quad (1)$$

where  $n$  is an integer and  $p$  is the momentum and the integral is taken over one complete period of the motion. The integer  $n$  is usually called as a quantum number.

The equation (1) necessarily leads to the quantization (i.e., discreteness) of energy. Let  $E_n$  be the energy for the  $n$ th orbit. When the particle changes its orbit, say from  $n$  to a lower orbit  $m$ , then it will lose an energy  $E_n - E_m$ . If that energy is carried by a single photon, then the photon should have the frequency

$$f = \frac{E_n - E_m}{h} \quad .$$

This relation is called Bohr frequency condition. In the absorption of light, the same relationship has to be satisfied as well. It is obvious that the orbit with lowest energy is absolutely stable. Therefore, the absence of a quantized orbit with lower energy prevents the electrons in atoms from radiating.

When Bohr have solved the Hydrogen atom problem, it has been seen that all lines in the emission spectrum of Hydrogen atom can be predicted. It can also give the correct frequencies for the singly ionized Helium atom, which is only different from Hydrogen by the fact that its nucleus contains two protons. These successes of the Bohr's theory in explaining the spectra led to its fast recognition.

Experiments carried out by Franck and Hertz (Nobel in 1926) provided a direct way of showing that the energy levels exist. In these experiments, electrons are accelerated to a known energy and passed through a low density gas. When electron energies are low, the electron-atom collisions are elastic. But if the electrons have sufficiently high energy, then inelastic collisions can occur. In inelastic collisions, part of the initial energy is converted to excite the atom (say from level-1 to level-2). In that case, it is observed that the current decreases and the gas starts to emit light.

Old quantum theory contained a number of unjustifiable assumptions and people knew that it was just an approximate form of a correct theory. But, with the lack of the correct one, they kept on using it until 1926. It is quite interesting to see that during this period, a lot of information is gained about the structure of atoms by investigating their spectra and using the old quantum theory. When the atoms are subjected to constant electric (Stark effect, Nobel 1919) or magnetic (Zeeman effect) fields, the energies and the spectral frequencies change. From these kind of experiments, they could understand the nature of quantum numbers. The spin of the electrons and their associated quantum numbers (Goudsmit and Uhlenbeck) and the exclusion principle of Pauli (Nobel 1945) are proposed during this period.

Let me mention one thing that is wrong with old quantum theory, a point that I will return later. Remember that to produce light with frequency  $f$ , there should be an oscillator with the same frequency. With the Bohr frequency condition, this implies that in  $n$  to  $m$  transition, there should be a charge oscillating with frequency  $f = (E_n - E_m)/h$ . The obvious candidate would be the periodic motion of electrons in periodic orbits. However, when you calculate the frequencies of this motion, you will see that the frequencies of  $n$ th or  $m$ th orbit are different from  $f$ . However, if you consider large successive quantum numbers ( $n + 1$  to  $n$  transition,  $n \gg 1$ ), then it can be shown that Eq. (1) leads to approximately matching values of all these three frequencies. There appears to be no problem for large quantum numbers, but an explanation is necessary for the small ones.

**Louis de Broglie, 1924.** A significant development was the PhD thesis of Louis de Broglie. In there, he suggested that there are waves accompanying each particle with a wavelength

$$\lambda = \frac{h}{p} \quad ,$$

where  $p$  is the momentum of the particle. He did not speculate on the nature of these waves (only later he proposed the guiding-wave interpretation). Experimental verification

that the electrons can show interference effects just like waves came subsequently. In 1925, Davisson and Germer's experiments looked at the diffraction of electrons reflected from crystal surfaces, in 1927 G.P. Thomson (son of J.J. Thomson) and his student diffracted them by thin films. Davisson and Thomson shared the Nobel Prize in 1937 "for their experimental discovery of the diffraction of electrons by crystals", de Broglie won the Prize in 1929 "for his discovery of the wave nature of electrons".

Using de Broglie's equation, the basic equation of old quantum theory, Eq. (1), and the energy levels of the Hydrogen atom can be obtained easily (this is the method used in introductory textbooks). If the electron is in a circular orbit with radius  $r$ , then momentum has constant magnitude.

$$m \frac{v^2}{r} = \frac{e^2}{r^2} \quad \longrightarrow \quad p = mv = \sqrt{\frac{me^2}{r}} \quad .$$

On the other hand, if the electron's wave does not destructively interfere with itself, then the circumference of the orbit should be an integer multiple of wavelength:  $2\pi r = n\lambda$ , where  $n$  is an integer. From here we can find the radius and the energy of the  $n$ th orbit as

$$\begin{aligned} r_n &= n^2 \frac{\hbar^2}{me^2} \quad , \\ E_n &= \frac{1}{2}mv_n^2 - \frac{e^2}{r_n} = -\frac{e^2}{2r_n} = -\frac{me^4}{2\hbar^2} \frac{1}{n^2} \quad . \end{aligned}$$

Here  $\hbar$  is

$$\hbar = \frac{h}{2\pi} \quad ,$$

which is also called Planck's constant. It appears to be the constant that simplifies the equations a lot, for this reason we will use it from now on.

This is not a satisfactory derivation either. If there is a wave accompanying the electron, then this wave is extended in space. It has a width and cannot be constrained to a constant radius  $r_n$ . Similarly, for particles whose momentum change from position to position, one has to take into account the changes in the wavelength. In short, a wave equation has to be formed.

**Erwin Schrödinger, 1926.** This equation is proposed by Schrödinger. Apparently, he first tried to form a relativistically correct equation first, but was dissatisfied with its results. Sometime later he formed the non-relativistic equation and decided to submit it for publication. Here we will give a sort of a derivation of the equation of the Schrödinger, but keep it in mind that it is not a derivation in the correct sense of the word. The main strength of the equation is its ability to predict the results of *all* experiments (with suitable relativistic generalizations of course) and not its derivation.

First start with a nonrelativistic particle in 1D having a Hamiltonian of the form

$$H = \frac{p^2}{2m} + V(x) \quad .$$

Remember that the Hamiltonian function in classical mechanics has the important job of describing the time development of particle's state through the equations

$$\frac{dx}{dt} = \frac{\partial H}{\partial p} \quad , \quad \frac{dp}{dt} = -\frac{\partial H}{\partial x} \quad .$$

But it has another important property that its value gives the energy. So, we will consider the motion of a particle with fixed energy  $E$ . What is the equation satisfied by de Broglie's wave knowing that the momentum changes with position according to

$$E = \frac{p^2}{2m} + V(x) \quad ?$$

Consider first a wave magnitude function  $\psi(x)$  having a fixed wavelength (constant momentum).

$$\psi(x) = A \sin \frac{2\pi}{\lambda} x = A \sin kx \quad .$$

Here  $k = 2\pi/\lambda$  is known as the wavenumber and de Broglie's relation appear as  $p = \hbar k$ . We see that the second derivative of  $\psi$  satisfies

$$\hbar^2 \frac{\partial^2 \psi}{\partial x^2} = -\hbar^2 k^2 \psi = -p^2 \psi \quad .$$

So, it appears that we can replace the square of momentum with the differential operator

$$p^2 \rightarrow -\hbar^2 \frac{\partial^2}{\partial x^2} \quad .$$

Since our energy expression contains  $p^2$ , we can write the wave equation as

$$E\psi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) \quad .$$

This is the time-independent Schrödinger equation. The equation also describes the change in the amplitude of the wave as well as its wavelength. An important feature of the equation is that it can leak into classically forbidden regions (regions where  $E < V(x)$ ). So, the wave will be present in regions where a classical particle would never visit. The extension to 3D is obvious: replace  $\vec{p}^2$  by  $-\hbar^2 \nabla^2$ .

**Time dependent.** Let us introduce the time dependence into the wave equation. For this, however, we have to introduce complex numbers into our equation. Consider a wave with fixed wavelength  $\lambda$  and fixed frequency  $f$ . The complex valued function

$$\psi(x, t) = A e^{i\left(\frac{2\pi}{\lambda}x - 2\pi ft\right)} = A e^{i(kx - \omega t)} \quad ,$$

describes such a wave. Here  $\omega = 2\pi f$  is called the angular frequency. The choice of sign of  $i$  is arbitrary of course, but with this choice we have the association of momentum with the differential operator

$$p \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial x} \quad .$$

For the energy-frequency relationship, we use Planck's formula (why?):  $E = hf = \hbar\omega$ . With this we can make the association

$$E \rightarrow i\hbar \frac{\partial}{\partial t} \quad .$$

Inserting all of this into the equation for energy we get the time dependent Schrödinger equation

$$i\hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + V(x)\psi(x, t) \quad .$$

Before going further, let us conclude the historical development. Before Schrödinger published his wave equation, Werner Heisenberg have developed his own equations that we now call as “matrix mechanics”. Both theories appeared to be describing correctly all physical problems that can be solved at that time. After a brief period of quarrel between these two people on which theory is correct, Schrödinger was able to show the equivalence of both theories. Heisenber won the Nobel Prize in 1932, and Schrödinger together with Dirac won it in 1933.

## Quantum Mechanics

How the Schrödinger equation is constructed from a given Hamiltonian function should be obvious. We replace various physical quantities by certain differential operators. Momentum is replaced by

$$\vec{p} \rightarrow \hat{p} = \frac{\hbar}{i} \vec{\nabla} \quad , \quad \hat{p}_x = \frac{\hbar}{i} \frac{\partial}{\partial x} \quad , \quad \hat{p}_y = \frac{\hbar}{i} \frac{\partial}{\partial y} \quad \text{etc.}$$

From now on I will use hats to represent operators. Using these replacements in the Hamiltonian function we get the Hamiltonian operator

$$\hat{H} = H(\hat{p}, \vec{r}) = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) \quad .$$

We can also consider the position as an operator with the identification

$$\hat{r} = \text{“multiply by” } \vec{r} \quad .$$

The Schrödinger equation is

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \quad . \tag{2}$$

We see that the Hamiltonian operator, just like the case in classical mechanics, contains the information for the time-development of the wave. The extension to more than one particle case is also obvious. In that case the wavefunction is a function of the coordinates of all the particles (plus time).

To solve the equation (2) we need to impose the boundary condition that the wavefunction goes to zero at infinities (when we see the interpretation of the wavefunction, we will see that the correct condition is square integrability). First, consider the eigenvalue equation

$$\hat{H}\varphi_n = E_n\varphi_n$$

which is nothing other than the time-independent Schrödinger equation with energy  $E_n$ . For bound states of particles, the eigenvalues of this equation will be discrete, the quantization of energy is the natural product of the wave equation. If the eigenfunctions  $\varphi_n$  form a complete set for the space of possible wavefunctions, then the general solution of Eq. (2) can be written as

$$\psi(\vec{r}, t) = \sum_n c_n e^{-iE_n t/\hbar} \varphi_n(\vec{r}) \quad .$$

Now, at this point we need to say something about the interpretation of the wavefunction. Schrödinger assumed that the charge density of an electron in a Hydrogen atom can be expressed as

$$\rho(\vec{r}, t) = (-e) |\psi(\vec{r}, t)|^2 \quad .$$

(This interpretation is not wrong for a single particle.) So we have a distribution of charge in space. The total charge should necessarily be  $(-e)$ , for this reason we have the condition

$$\int |\psi(\vec{r})|^2 d^3\vec{r} = 1 \quad .$$

In other words,  $\psi$  should be square integrable and moreover its norm should be equal to 1.

First consider the case where  $\psi$  is formed by a single eigenfunction of  $\hat{H}$ , say the one with quantum number  $n$ . The wavefunction is

$$\psi(\vec{r}, t) = \varphi_n(\vec{r}) e^{-iE_n t/\hbar} \quad .$$

In that case the charge density at time  $t$  is

$$\rho(\vec{r}, t) = (-e) |\varphi_n(\vec{r})|^2 \quad ,$$

In other words it will be time-independent. As a result, there won't be any radiation emitted either. What prevents the Hydrogen atom from collapsing is that lack of time dependence. Because of this, the states represented by the eigenfunctions of the Hamiltonian are called "stationary states".

Now consider a wavefunction which is a superposition of two eigenstates of the Hamiltonian, say the  $n$ th and  $m$ th levels. In that case the wavefunction is like

$$\psi(\vec{r}, t) = a\varphi_n(\vec{r})e^{-iE_n t/\hbar} + b\varphi_m(\vec{r})e^{-iE_m t/\hbar} \quad .$$

If we calculate the charge density, this time we find a time-dependent expression,

$$\begin{aligned} \rho(\vec{r}, t) &= (-e) |\psi(\vec{r}, t)|^2 \\ &= (-e) (|a|^2 |\varphi_n(\vec{r})|^2 + |b|^2 |\varphi_m(\vec{r})|^2) + \\ &\quad (-e) a^* b \varphi_n(\vec{r})^* \varphi_m(\vec{r}) \exp\left(i \frac{E_n - E_m}{\hbar} t\right) + c.c. \\ &= \rho_{av}(\vec{r}) + \rho_1(\vec{r}) \cos\left(\frac{E_n - E_m}{\hbar} t\right) + \rho_2(\vec{r}) \sin\left(\frac{E_n - E_m}{\hbar} t\right) \end{aligned}$$

In fact it is a periodic function of time. So, an EM wave will be produced and the frequency of the wave will be given by the Bohr frequency condition

$$f = \frac{1}{2\pi} \frac{E_n - E_m}{\hbar} = \frac{E_n - E_m}{h} .$$

As a result, this frequency condition can be naturally derived from the theory. Moreover, the polarization of the wave (photon) that is produced can be determined from the charge density expression given above.

**The Probabilistic Interpretation:** Apart from the equation for the wavefunctions, we also need a correct interpretation of what it means. This is important for experimenters that need to relate the concepts of the theory to what they observe, but it is also important for understanding the workings of nature. The interpretation developed by Max Born in 1926 (Nobel Prize, 1954) and then subsequently advocated by Niels Bohr and his students in Copenhagen has been satisfactory for the experimenters, but opposed fiercely by many others. I will try to describe below this interpretation (often called as the Copenhagen interpretation to distinguish it from other kinds of interpretations) without mentioning other possibilities.

What Max Born has noticed that the experimenters see the electron as a particle. Consider an electron scattering experiment where an electron is thrown towards a stationary target atom. In such an experiment, the waves represented by the wavefunction  $\psi$  will disperse in all possible directions after meeting the target. So, if the wave picture was exactly correct, then we would see waves reaching to all detectors placed at different positions. However, what the experimenters see is that at most one detector fires up and shows the presence of the electron. So, although the wavefunction is necessary to describe the behavior of the electrons correctly, the electron was still behaving like a particle in certain cases. So, somehow these two distinct features formed a consistent whole.

Max Born then proposed that the wavefunction  $\psi$  represents the probability for the various properties of the particles. The Schrödinger equation then shows how this probability propagates in space. He proposed that the probability of finding the particle at position  $\vec{r}$  inside a volume  $\Delta V$  is given by  $|\psi(\vec{r}, t)|^2 \Delta V$ . In that case,  $|\psi(\vec{r}, t)|^2$  has to be called as the probability density. The complex number,  $\psi(\vec{r}, t)$  is frequently referred as probability amplitude. Since the total probability has to be 1, we have the condition

$$\int |\psi(\vec{r}, t)|^2 d^3\vec{r} = 1 .$$

Now, it appears that the admissible wavefunctions must be in the Hilbert space of square-integrable functions and only those ones with norm 1 has to be used. The Hilbert space structure appears to be very important when we consider the other physical properties of the particles, like the energy. First note that the Hamiltonian is a hermitian (self-adjoint) operator. For this reason, its eigenfunctions form a complete orthonormal basis for the Hilbert space if they are also chosen as normalized,

$$\langle \varphi_n | \varphi_m \rangle = \int \varphi_n(\vec{r})^* \varphi_m(\vec{r}) d^3\vec{r} = \delta_{nm} = \begin{cases} 0 & \text{if } n \neq m \text{ ,} \\ 1 & \text{if } n = m \text{ .} \end{cases}$$

(Note the way we define the inner product.) In that case we can expand any initial state  $\psi(\vec{r}, 0)$  in terms of eigenfunctions of  $\hat{H}$  as follows

$$\psi(\vec{r}, 0) = \sum_n c_n \varphi_n(\vec{r}) \quad , \quad \text{where} \quad c_n = \langle \varphi_n | \psi(0) \rangle \quad .$$

The wavefunction at time  $t$  will then be

$$\psi(\vec{r}, t) = \sum_n c_n \varphi_n(\vec{r}) e^{-iE_n t / \hbar} \quad .$$

If we check the normalization condition we find

$$\int |\psi(\vec{r}, t)|^2 d^3\vec{r} = \sum_n |c_n|^2 = 1 \quad .$$

This equation tells us that if the initial wavefunction  $\psi(\vec{r}, 0)$  has norm 1, then after solving the Schrödinger equation, Eq. (2), we get a wavefunction  $\psi(\vec{r}, t)$  which has norm 1 at all times. The Schrödinger equation describes a unitary time evolution consistent with the probabilistic interpretation.

To Born, what was striking about the equation

$$\sum_n |c_n|^2 = 1 \quad ,$$

was that it looked like an equation which says “total probability is 1”. For this reason, he interpreted  $|c_n|^2$  as the probability of measuring the energy of the particle to be  $E_n$ .

**Any Observable.** We can extend this idea to other possible properties of the particle. Consider any property  $A$  of the particle that can be measured by some experimental device. There should be a corresponding operator  $\hat{A}$  acting on the Hilbert space of wavefunctions. This operator has to be Hermitian necessarily due to the reasons we will see below. Similar to what we have done above, we first need to solve the eigenvalue equation for the operator  $\hat{A}$ ,

$$\hat{A}\alpha_n(\vec{r}) = \lambda_n \alpha_n(\vec{r}) \quad .$$

The eigenvalues  $\lambda_n$  are going to be interpreted as the quantized values of observable  $A$ . For this reason they have to be real numbers (no experimental device can measure a complex value).

Second, the eigenfunctions  $\alpha_n(\vec{r})$  should have the capability of forming an orthonormal basis for the Hilbert space. These two conditions imply that  $\hat{A}$  has to be hermitian. In any case, if the particle has the wavefunction  $\psi(\vec{r})$  at the time the measurement is taken, we need to make the following expansion

$$\psi(\vec{r}) = \sum_n d_n \alpha_n(\vec{r}) \quad , \quad \text{where} \quad d_n = \langle \alpha_n | \psi \rangle \quad ,$$

where we have used the orthonormality property  $\langle \alpha_n | \alpha_m \rangle = \delta_{nm}$ . Then  $|d_n|^2$  is the probability of measuring  $A$  to be  $\lambda_n$ . Since the total probability is  $\langle \psi | \psi \rangle = \sum_n |d_n|^2 = 1$  we have covered all possibilities.

An important quantity that we would like to work with is the average value of measurements. We suppose that the particle is repeatedly prepared in the state represented by the wavefunction  $\psi(\vec{r})$  and then the measurement of  $A$  is carried out. The statistical average of the results obtained is represented by  $\langle A \rangle$  and is frequently called as the “expectation value”. It can be expressed as

$$\langle A \rangle = \sum_n \lambda_n |d_n|^2 = \sum_{nm} d_n^* \lambda_m d_m \langle \alpha_n | \alpha_m \rangle = \langle \psi | \hat{A} \psi \rangle = \langle \psi | \hat{A} | \psi \rangle \quad .$$

This expression is very convenient because to calculate it we don't need to solve the eigenvalue equation. We just need to know how to apply  $\hat{A}$  on  $\psi$ . It can also be extended to the average of the square of observed values

$$\langle A^2 \rangle = \sum_n \lambda_n^2 |d_n|^2 = \langle \psi | \hat{A}^2 \psi \rangle = \langle \psi | \hat{A}^2 | \psi \rangle \quad .$$

A very simple proof that  $\hat{A}$  is hermitian can be given if it is postulated that the expectation value can be calculated as  $\langle A \rangle = \langle \psi | \hat{A} \psi \rangle$  (without specifying the probabilities, that postulate alone is not sufficient, but we continue). Since the experimenters can only measure real numbers, the expectation value has to be real as well. If the wavefunction is  $\psi$ , we have

$$\langle A \rangle^* = \langle \psi | \hat{A} \psi \rangle^* = \langle \hat{A} \psi | \psi \rangle = \langle \psi | \hat{A}^\dagger \psi \rangle \quad ,$$

where  $\hat{A}^\dagger$  is the hermitian conjugate of the operator  $\hat{A}$ . From here, we get

$$\langle A \rangle - \langle A \rangle^* = \langle \psi | (\hat{A} - \hat{A}^\dagger) \psi \rangle = 0 \quad .$$

Next we claim that all possible normalized wavefunctions are physically realizable wavefunctions for the particle. Then the equation above says that, for all functions with norm 1, the expectation value of  $\hat{A} - \hat{A}^\dagger$  is 0. It is then a straightforward exercise in Hilbert space theory to show that this implies  $\hat{A} - \hat{A}^\dagger = 0$ , i.e.,  $\hat{A}$  is hermitian.

**Momentum.** Momentum is also a possible observable. We have said before that it is represented by the operator (I will consider the  $x$ -component of momentum)

$$\hat{p}_x = \frac{\hbar}{i} \frac{\partial}{\partial x} \quad .$$

It can be shown quite easily that it is hermitian,

$$\begin{aligned} \langle \phi_1 | \hat{p}_x \phi_2 \rangle &= \int \phi_1^* \frac{\hbar}{i} \frac{\partial \phi_2}{\partial x} d^3 \vec{r} \\ &= \frac{\hbar}{i} \int \frac{\partial (\phi_1^* \phi_2)}{\partial x} d^3 \vec{r} - \frac{\hbar}{i} \int \frac{\partial \phi_1^*}{\partial x} \phi_2 d^3 \vec{r} \\ &= 0 + \left( \frac{\hbar}{i} \int \frac{\partial \phi_1}{\partial x} \phi_2^* d^3 \vec{r} \right)^* \\ &= \langle \phi_2 | \hat{p}_x \phi_1 \rangle^* = \langle \hat{p}_x \phi_1 | \phi_2 \rangle \\ &= \langle \phi_1 | \hat{p}_x^\dagger \phi_2 \rangle \quad , \end{aligned}$$

which implies that  $\hat{p}_x = \hat{p}_x^\dagger$ . In here we have used the square integrability property to deduce that  $\phi_1^* \phi_2$  goes to zero in the limit  $x \rightarrow \pm\infty$ .

What about the eigenfunctions? It can be seen that the function  $e^{ikx}$  is an eigenfunction of  $\hat{p}_x$  with eigenvalue  $\hbar k$ . However,  $e^{ikx}$  is not in the Hilbert space. Physicists are not overwhelmed by such “details” and proceed to treat these functions as if they were in Hilbert space. For our purposes, we can consider the following Fourier transform

$$\psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int \phi(\vec{k}) e^{i\vec{k}\cdot\vec{r}} d^3\vec{k} \quad .$$

We then note the Parseval’s identity

$$\int |\psi(\vec{r})|^2 d^3\vec{r} = \int |\phi(\vec{k})|^2 d^3\vec{k} = 1 \quad .$$

Seeing this equation as the way Born did, we can interpret  $|\phi(\vec{k})|^2$  as the probability density for momentum distribution. In other words, the probability of measuring the momentum to be  $\hbar\vec{k}$  within a “ $\vec{k}$ -volume” of  $\Delta V_k$  is  $|\phi(\vec{k})|^2 \Delta V_k$ . The function  $\phi(\vec{k})$  is called the momentum-space wavefunction.

**Lack of Determinism.** We have talked about probabilities above, but we haven’t said anything about how each individual outcome of measurements occur. The previously seen notions of probability in physics have always been statistical. The outcomes of experiments are not random, but since we cannot precisely measure the initial conditions of physical systems, the outcomes *appear* random to us. So the probabilities actually reflect our lack of knowledge about the system in question. If we knew a lot about the system we can predict the outcomes. For example in coin flip experiments, you need to know the exact value of impulse you give to the coin and the exact place you hit to be able to determine if it will end up heads or tails. Most of the time, the system is chaotic so that uncertainties in initial values prevent you from making a prediction. So, you are stuck with the probabilities.

In quantum mechanics however, the concept of probability is included at its roots, distinct from the classical notion of probability. Consider the measurement of the observable  $\hat{A}$ . I am going to use the notation above for eigenfunctions. Suppose the wavefunction of the particle is  $\psi(\vec{r})$  and its expansion is

$$\psi(\vec{r}) = \sum_n d_n \alpha_n(\vec{r}) \quad .$$

We have said that measurement of  $A$  yields the eigenvalue  $\lambda_n$  with probability  $|d_n|^2$ . If there were really a way to determine this outcome (namely  $\lambda_n$ ), then this information is not contained in the wavefunction,  $\psi(\vec{r})$ . If you insist that the outcomes are realized deterministically, then you need to invent new variables other than the wavefunction. However, the experiments carried out up to now shows us that only the wavefunction and the Schrödinger equation is necessary to explain all of them. You have two options: You either extend the theory and introduce new variables (these are called Hidden Variable Theories) or you stick with the present status and accept non-deterministic aspect of it.

The Copenhagen interpretation follows the second one. As a result, before actually measuring  $A$ , you have no way of predicting which outcome will be the obtained. Also, any one of them with nonzero  $d_n$  can really occur. During the experiment, nature *some-how* decides which one should appear. This lack of determinism terrified a lot of people. Einstein was their leader. Bohr was the defender of the interpretation. After a lot of discussion, this view gained weight. But even today there are serious works concentrating on other possible interpretations.

**Collapse.** To complete the theory, we have to mention one last feature. Consider an experiment where you measure  $A$ . After some indeterministic measurement process we get one particular result,  $\lambda_n$ . Now, suppose that we measure  $A$  again immediately after the first measurement. We need to do this immediately because with time, the wavefunction could change. Normally we should obtain exactly the same result. In other words, both experiments should give the same value  $\lambda_n$ . For a correct measurement concept we need to have this feature. For this reason, the second experiment has no uncertainty in it. If this is so, then the first measurement of  $A$  should have caused a discontinuous change in the wavefunction to the eigenfunction of  $\hat{A}$  corresponding to  $\lambda_n$ .

To summarize, suppose that you make a measurement at time  $t = t_{meas}$ . Prior the measurement, the wavefunction is

$$\psi(\vec{r}, t = t_{meas} - \epsilon) = \sum_m d_m \alpha_m(\vec{r}) \quad ,$$

which can be anything. If the measurement yields  $A = \lambda_n$ , then the wavefunction just after the measurement has to be

$$\psi(\vec{r}, t = t_{meas} + \epsilon) = \alpha_n(\vec{r}) \quad .$$

In other words, the effect of the measurement on the wavefunction is a projection to an eigenfunction (or eigenspace) of  $\hat{A}$  and re-normalization. This is called the collapse of the wavefunction.

It can be seen that the measurement introduces an unavoidable change in the wavefunction. A change that destroys all information that is carried by the wavefunction before the measurement. So, measurement in quantum mechanics does not have the conventional meaning of “learning”, its meaning would be more like “changing and pretending that you have learned”.

**State.** In quantum mechanics, the wavefunction  $\psi(\vec{r}, t)$  contains all information that you can ever learn about the particle. For this reason, it is frequently referred as the *state*. For example, if you know the precise state at a certain time, then you can calculate the state at any other time by integrating the Schrödinger equation. The corresponding notion of state in classical mechanics would be its position and momentum:  $(\vec{r}, \vec{p})$ , which is a point in the six dimensional phase space. In quantum mechanics, however, the state space becomes an infinite dimensional Hilbert space.

Consider now the position property of particle in a state  $\psi(\vec{r})$ . Since the wavefunction is distributed in space, there is no single definite position which we can say the particle is

located. Measurement of position gives us only one of these possibilities, but before the measurement each position is a possibility.

There is an interpretation which you might hear a lot which goes like this: “Particle is actually somewhere but we don’t know where it is”. This sentence actually assumes that state= $(\psi, \vec{r}_{real})$ , where  $\vec{r}_{real}$  is the supposed real position of particle. In other words, it assumes that there are more things to know about the particle than the wavefunction. This is a hidden variable theory and is entirely different from quantum mechanics.

In quantum mechanics we might state the same thing as “particle is everywhere” which, at first sight, might look confusing. Another alternative statement is “there is no meaning of question ‘where is it?’ without actually measuring it”. It appears that the classical notions of *definite position* and *definite momentum* cannot be directly carried over to quantum mechanics. We have notions of position and momentum in quantum mechanics but their nature is different from our classical notions.

This situation is similar to the notion of *absolute time* we meet in relativity. Absolute time is a concept which *appears* to be true in the non-relativistic limit. Of course such a notion is invalid, and we will make a lot of mistakes if we try to directly carry it over to relativistic problems. Same in quantum mechanics. We should, then, get rid of the notions of *definite* values of some mechanical quantities.

**Uncertainties.** We don’t have definite values of position and momentum, but there is also a limitation on how close we can get to definiteness. A mathematical measure of this is the standard deviation of measurement results which is frequently called uncertainty. For example, the uncertainty in  $x$ -component of position is

$$\Delta x^2 = \langle (x - \langle x \rangle)^2 \rangle = \int |x - \langle x \rangle|^2 |\psi|^2 \quad .$$

Remember that this gives the deviation of measurement results from the average in a series of repeated measurements on the same state  $\psi$ . In other words, each time the particle has to be re-prepared in the same state. Uncertainty in momentum is defined similarly,

$$\Delta p_x^2 = \langle (p_x - \langle p_x \rangle)^2 \rangle \quad .$$

Now, it appears that the position and momentum operators corresponding to the same component do not commute with each other. They have a commutator

$$\hat{x}\hat{p}_x - \hat{p}_x\hat{x} = i\hbar \quad .$$

This commutation relation implies that the product of respective uncertainties has a lower bound. The proof goes like this. First note that

$$\begin{aligned} \Delta x^2 &= \langle \psi | (\hat{x} - \langle x \rangle)^2 | \psi \rangle \\ &= \langle (\hat{x} - \langle x \rangle) \psi | (\hat{x} - \langle x \rangle) \psi \rangle \quad . \end{aligned}$$

Let us define two vectors in the Hilbert space by

$$\begin{aligned} \phi_1 &= (\hat{x} - \langle x \rangle) \psi \\ \phi_2 &= (\hat{p}_x - \langle p_x \rangle) \psi \end{aligned}$$

In that case, the uncertainties can be expressed as norms of these vectors:  $\Delta x = \|\phi_1\|$  and  $\Delta p_x = \|\phi_2\|$ .

At this point we can use Schwarz inequality

$$\Delta x \Delta p_x = \|\phi_1\| \cdot \|\phi_2\| \geq |\langle \phi_1 | \phi_2 \rangle| \quad .$$

Now, it appears that we can calculate the imaginary part of the inner product  $\langle \phi_1 | \phi_2 \rangle$  as

$$\begin{aligned} \text{Im} \langle \phi_1 | \phi_2 \rangle &= \frac{1}{2i} (\langle \phi_1 | \phi_2 \rangle - \langle \phi_2 | \phi_1 \rangle) \\ &= \frac{1}{2i} [ \langle (\hat{x} - \langle x \rangle) \psi | (\hat{p}_x - \langle p_x \rangle) \psi \rangle - c.c. ] \\ &= \dots \\ &= \frac{1}{2i} \langle \psi | (\hat{x} \hat{p}_x - \hat{p}_x \hat{x}) \psi \rangle \\ &= \frac{\hbar}{2} \quad . \end{aligned}$$

Since  $|\langle \phi_1 | \phi_2 \rangle|$  is greater than its imaginary part, we have

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad .$$

This is called the Heisenberg uncertainty relation.

Note that it is a relation about the same state  $\psi$  for possible position or momentum measurements which have not been carried out. It tells you that you cannot prepare a state where you can choose uncertainties in position and momentum to be as small as you wish. It also tells you that you cannot measure position and momentum at the same time ( $\Delta x$ ,  $\Delta p_x$  in collapsed state). If you have measured the position with a very small uncertainty, then the equation tells you that the uncertainty in momentum have got larger (in the collapsed state again).

**Postulates.** Now we need to generalize what we have said above about the basic features of quantum mechanics to other possibly more complicated cases. The case of  $N$  particles seems simple, you just need to consider complex-valued square integrable functions of  $N$ -positions ( $3N$  real variables),  $\psi(\vec{r}_1, \dots, \vec{r}_N, t)$ . There are cases where the number of particles can change, which is the situation particle physicists meet a lot. For example, in a neutron decay, a neutron is converted into three distinct particles. So, you should be able to use  $N = 1$  particle wavefunctions together with  $N = 3$  particle wavefunctions. There is also the spin degree of freedom of electrons which needs only a two dimensional Hilbert space. In each of these cases the Hilbert space and the operators corresponding to observables have to be constructed. But the basic machinery of quantum mechanics remains the same. I am going to state these as two postulates, first one being universally accepted and the second one being the controversial one.

**Postulate 1: States.** For every isolated physical system, there is a separable, complex Hilbert space describing the states of the system, such that

- Every normalized vector,  $|\psi\rangle$ , represents a state
- To every physically realizable state there is a normalized vector that represents it
- Overall phase factors do not change the state (in other words  $|\psi\rangle$  and  $e^{i\theta}|\psi\rangle$  represent the same state)  
(The three statements above basically tells us that there is a one-to-one correspondence between states and rays in the Hilbert space.)
- There is a linear, unitary time-evolution operator  $\hat{U}(t_2, t_1)$ , which, when acts on the state at time  $t_1$  gives the state at time  $t_2$ . (In other words, if  $|\psi(t)\rangle$  is the state of the system at time  $t$ , then  $\hat{U}(t_2, t_1)|\psi(t_1)\rangle = |\psi(t_2)\rangle$ .)

**Postulate 2: Observables.** For every measurable quantity  $A$ , there is a corresponding hermitian operator  $\hat{A}$  such that

If  $\lambda_n$  are its eigenvalues and  $|\alpha_n\rangle$  are its eigenvectors chosen such that they form an orthonormal basis

$$\hat{A}|\alpha_n\rangle = \lambda_n|\alpha_n\rangle \quad , \quad \langle\alpha_n|\alpha_m\rangle = \delta_{nm} \quad ,$$

and if  $A$  is measured when the system is in state  $|\psi\rangle$ ,

- The result is one of the eigenvalues,  $\lambda_n$ ,
- the probability of that result is  $p_n = |\langle\alpha_n|\psi\rangle|^2$ ,
- and the state collapses to  $|\alpha_n\rangle$  after the measurement.

The Schrödinger equation itself can be obtained from the time evolution operator where we define the Hamiltonian at time  $t$  by

$$i\hbar\frac{\partial}{\partial t}\hat{U}(t, t_1) = \hat{H}(t)\hat{U}(t, t_1) \quad ,$$

which implies that  $\hat{H}(t)$  is hermitian.

**Schrödinger's Cat.** An interesting feature of the first postulate is that it allows us to take linear combinations of a number of states and in this way construct new, physically realizable states. The linear combinations of vectors are usually called superpositions, a word borrowed from wave phenomena. The possibility of forming superpositions is the most important distinctive feature of quantum mechanics that separates it from the classical ones.

A strange example is proposed by Schrödinger. He also showed how such states can be formed in practice. Consider a cat as a physical system. We know that there is a Hilbert space that describes all possible states of that cat. Let  $|A\rangle$  be one particular state where

the cat is alive. Let  $|D\rangle$  be another state where the cat is dead. Postulate 1 tells us that there is a state of the cat represented by the following vector of the Hilbert space,

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|A\rangle + |D\rangle) \quad .$$

There are a number of things which is strange about this state. The cat is neither alive nor dead, it is in a strange state foreign to our classical senses. We might also say that it is “both dead and alive” at the same time.

Of course, to keep the cat in that state, you have to isolate it inside a box because seeing it amounts to a measurement (and measurement means collapse, we don't want that). Also, the walls of the box should be perfectly isolated because hearing the voice of the cat (or not hearing it) also amounts to a measurement. Can such states be really prepared? The postulates of quantum mechanics does not state anything like “these laws do not apply to such and such bodies”. So, it appears that, all macroscopic objects as well as all microscopic ones should obey quantum mechanics (this is also the basic difficulty with postulate 2 where it is assumed that system is quantum mechanical but the experimenter is classical, but that is another matter).

Now, if we open the box and measure “what the cat is doing” then the state will collapse to the state  $|A\rangle$  or to  $|D\rangle$ , i.e., we will see it as either alive or dead. It appears that with the means available to us, we cannot do any other measurement, so we will always cause the destruction of the state  $|\psi\rangle$ . However, it is possible to prepare sufficiently macroscopic objects in such superposition states. In an experiment carried out a few years ago, SQUIDS which contain billions of electrons, are made to enter into a superposition of two macroscopically distinct states. In that experiment, however, they had a method to determine whether the device is in the superposition state or not (in other words, there was an observable with  $|\psi\rangle$  being the eigenvector). Is it possible to construct such an observable (an experimental technique) that can show that the cat really is in the state  $|\psi\rangle$  and not in  $|A\rangle$  or  $|D\rangle$ ? Until we answer this question, we cannot answer what the cat is really doing. But, it appears that, there is no fundamental macroscopic limit on the applicability of quantum mechanics.