Question I (20 pnts.): What is the net electric force on a unit positive charge placed at the center of a square of side $b$ which has charges $q, 2q, -4q$ and $2q$ placed at the four corners?

Question II (20 pnts.): The neutral hydrogen atom in its normal state behaves in some respects like an electric charge distribution which consists of a point charge of magnitude $+e$ surrounded by a distribution of negative charge whose density is given by $\rho(r) = -Ce^{-2r/\alpha}$. $C$ is a constant required to make the hydrogen atom neutral.

(a) What is $C$?
(b) Find the net electric charge inside a sphere of radius $a_0$.
(c) What is the electric field strength in the entire space?

Question III (20 pnts.) An infinite plane has a uniform surface charge distribution $\sigma$ on its surface. Right next to it (see figure) is an infinite parallel layer of charge of thickness $d$ and uniform charge density $\rho$. All charges are fixed. Find the electric field everywhere (including the region inside the thick layer). Regardless of what it might look like on the figure, assume that the surface with charge distribution $\sigma$ has zero thickness.

Question IV (20 pnts.) Evaluate the following vector expressions. They are each worth 4 pnts.

(a) $\nabla \cdot \vec{V}$ for $\vec{V}(x, y, z) = xy^2 \hat{i} + \frac{y}{x} \hat{j} + (x^3 + yz + xz^2) \hat{k}$

(b) $\nabla \times \vec{V}$ for the same vector field in part (a).

(c) Gradient of $f(x, y, z) = x^3y + \ln(z + y) + e^y$

(d) Prove that $\nabla \times \nabla \phi = 0$ for a general function $\phi$.

(e) Prove that $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

Question V (20 pnts.) Read Chapter II-9-10 in Feynman’s Lectures on lightning and thunderstorms. Making this your starting point, write an essay on the mechanism of thunderstorms with appropriate references (no Internet references PLEASE). The work must be written IN YOUR OWN WORDS. It doesn’t matter if the English is bad, it will be corrected by your T.A. The more extensive is the research the higher will be your grade. The essay should
not exceed three typed pages (pt 10) but should not be less than one page. It is preferable that you TYPE your essay on a computer; hand-written essays are discouraged.

[1] Here $a_0$ is the quantum mechanically averaged radius of the electron orbit in the hydrogen atom in its ground (lowest energy) state