Homework II : More vectors, electric field, Gauss’ Law, conductors, electrostatic potential and energy, capacitors

**Question I (15 pnts.)** : Let $\vec{r}$ be a vector from some fixed point $(x_0, y_0, z_0)$ to the point $(x, y, z)$ and let $r$ be its length. Determine the following :

(a) $\vec{\nabla} (\vec{r} \cdot \vec{r})$

(b) $\vec{\nabla} (1/r)$

(c) What is the general formula for $\vec{\nabla} (r^n)$?

**Question II (15 pnts.)** : A thin rod of charge density $\lambda$ extends from $-d$ to $d$ along the $x$-axis and lies in the $xy$ plane.

(a) Find the potential at point $(2a, 0)$ along the $x$-axis (not on the line itself).

(b) Find the points on the $y$-axis that is at the same potential as the point in part (a).

**Question III (15 pnts.)** : A small charged sphere of mass $m$ and charge $q$ hangs from a silk thread that makes an angle $\theta$ with a large, flat uniformly charged insulating surface as seen in the figure. Find the surface charge density $\sigma$.

**Question IV (15 pnts.)** : A conducting cylinder of radius $r$ that is infinite in length has a uniform volume charge density of $\rho$. With the help of Gauss’ Law determine the potential at all points with respect to infinity. Sketch your result on an appropriate graph.

**Question V (15 pnts.)** : An infinite conducting cylinder of radius $r$ with a surface charge density of $\lambda$ C/m (watch the units!) is fixed at a distance $h$ above a grounded conducting infinite plane ($h$ is the perpendicular distance between the center of the cylinder and the plane and needless to say $h > r$). Using the method of image charges find

(a) The potential difference between the plate and the cylinder.

(b) The charge distribution on the plane (you may assume that the conducting plane lies on the $xy$-plane).

**Question VI (15 pnts.)** : Ink-jet printers are examples of devices that utilize charged droplets of liquid. The drop size is controlled by the specific charge (coulomb per kg) carried by the fluid.

(a) When a charged droplet splits into two parts, electrostatic repulsion drives the droplets apart and the electrostatic energy decreases. Calculate the decrease of electrostatic energy when a droplet of radius $R$ carrying a charge $Q$ splits into equal-sized droplets of charge $Q/2$. Assume that the distance by which they are separated is large compared to their new radius.

(b) The energy associated with surface tension is equal to a constant that is characteristic of the liquid, multiplied by the surface area. The constant is called surface tension of the liquid. The surface tension of water is $7.275 \times 10^{-2}$ J/m$^2$. When a droplet splits, the surface area increases and the energy associated with the surface tension increases proportionately. Calculate the gain in this energy when a droplet of radius $R$ splits into two equal sized droplets.
(c) A droplet of water has a radius of one micrometer and carries a specific charge of one C per kg. Will the droplet split?

(d) What is the radius of the largest droplet of water that is stable with this specific charge?

**Question VII (15 pnts.)**: Charge is uniformly distributed (density $\sigma_s$) over an infinite (in the x-direction) strip of width $w$. Determine the electric field at a point $P$ which is on a line perpendicular to the strip and at a distance $z$ from the center of the strip.

**Question VIII (15 pnts.)** *(Ohanian Chapter 27, Problem 28)* A spherical capacitor consists of two concentric spheres of metal of radii $R_1$ and $R_2$ as seen in the figure. The space between these spheres is filled with two kinds of dielectric. The dielectric in the upper hemisphere has a constant $\kappa_1$ and the dielectric in the lower hemisphere has a constant $\kappa_2$. What is the capacitance of this device?