Homework IV-V : Magnetic Fields, Induction, RLC Circuits, EM Waves

**Question I (25 pnts.)** : (Ohanian Chapter 32, Problem 39) A toroid with a square cross section has an inner radius of $R_1$ and an outer radius of $R_2$. The toroid has $N$ turns of wire carrying a current $I$. Assume that $N$ is very large so that the current may be regarded as uniformly distributed over the surface of the toroid.

(a) Find the magnetic field as a function of the radius.

(b) Find the flux passing through one turn and find the self-inductance of the toroid.

**Question II (30 pnts)** : Consider the setup seen in the picture below where two infinitely long wires carrying the same current $I$ in opposite directions are located a distance $d$ apart. A rectangular loop of dimension $a \times b$ is approaching the wires (all three objects are on the same plane) with a uniform velocity $v$. Find the induced emf in the loop.

**Question III (30 pnts.)** : The circuit seen in the figure below has a resistor $R$ and a movable bar of mass $m$. At the location of the circuit there is a uniform magnetic field of magnitude $B$ into the page. At time $t = 0$ the bar is thrust to the left with initial velocity $v_0$.

(a) Find the velocity as a function of time.

(b) How much distance does the rod cover before stopping?

**Question IV (30 pnts.)** : Imagine the same setup as that in Question II but this time replace the resistor with an inductor of inductance $L$. Answer the same questions referring to the figure below.

**Question V (30 pnts.)** : The RLC circuit seen in the picture above has its elements connected in parallel rather in series. If the initial conditions are such that the capacitor is fully charged and is just beginning to discharge, find the current in each branch as a function of time using the techniques derived in class.

**Question VI (30 pnts.)** : An alternating current $I_c = I_0 \cos(\omega t)$ flows down a long straight wire and back along a coaxial conducting cylinder of radius $R$.

(a) In what direction does the induced electric field point and what is its time-dependent magnitude?
(b) Find the displacement current density $\vec{J}_d$ and integrate it to get the displacement current.

(c) What is the ratio of $I_d$ to $I_c$? How high would the frequency be for $I_d$ to be one per cent of $I_c$?

**Question VII (25 pnts.)**: (Ohanian Chapter 36, Problem 8 with a modification) An electromagnetic wave traveling along the $x$ axis consists of the following superposition of two waves polarized along the $y$ and $z$ directions, respectively:

$$\vec{E} = \hat{y}E_0 \sin (\omega t - \omega x/c) + \hat{z}E_0 \cos (\omega t - \omega x/c)$$

Such an electromagnetic wave is said to be *circularly polarized*.

(a) Explain why such a name may be appropriate for such a wave.

(b) Calculate the instantaneous Poynting vector for this wave.