Homework II : Partial differential equations and traveling waves on strings

Guidelines for Homework III : Please read carefully!
1. Homework II is due Wednesday, 08/12 by 17:30.
2. Homework will be submitted via email to homework.phys343@gmail.com. Please send the Octave functions and scripts as attachments.

Reading assignment : Read Chapter 6 of Giordano and Nakanishi and the fifth lecture notes.

Question 1 (30 points): The van der Pol oscillator, which appears in electronic circuits, is described by the equation
\[ \frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} + \omega^2 x = 0. \]

Write an Octave script, oscillator_vdp.m that solves for \( x \) as a function of \( t \) and makes a plot showing both \( x \) and \( \frac{dx}{dt} \) as a function of time on the same plot, from \( t = 0 \) to \( t = 20 \) sec. For the parameters, use \( \omega = 1 \) and \( \mu = 1 \) and initial conditions \( x = 1 \) and \( \frac{dx}{dt} = 1 \).

Question 2 (35 points): The great mathematician Steven Strogatz states a definition of chaos as follows: "Chaos is aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions." In this example, you will take two oscillators that start from very close initial angles \( \theta_0,1 \) and \( \theta_0,2 \) and evolve according to the full oscillator equations (not the small angle but the regular, large angle treatment). Starting from the nonlinear oscillator program that we wrote in class, write an Octave script two_oscillators.m that describes the motion of two oscillators that start out with a difference in initial angle by 0.1 radians. Your program should plot the logarithm of the difference in angles of the two oscillators as a function of time. In the non-chaotic regime, the difference should always get smaller. In the chaotic regime, it should get larger and act in an unpredictable way. I suggest you take a look at the section in the book on this problem to get a better idea.

Question 3 (35 points): Copy your function propagate.m and your script string_fixed.m into new files called propagate_stiff.m and string_fixed_stiff.m respectively to write a simulation of a stiff string. The string that we studied in class was an ideal string that did not have any resistance to bending. A real, stiff string, however, displays a certain resistance and the associated wave equation is given by
\[ \frac{\partial^2 y}{\partial x^2} = c^2 \left( \frac{\partial^2 y}{\partial x^2} - \epsilon L \frac{\partial^4 y}{\partial x^4} \right) \]
which results in the discrete equation of motion
\[ y(i, n+1) = [2-2r^2-6\epsilon r^2 M^2]y(i, n) - y(i, n-1) + r^2(1+4\epsilon M^2)[y(i+1, n) + y(i-1, n)] - \epsilon r^2 M^2[y(i+2, n) + y(i-2, n)]. \]
where M is the number of partitions taken along the string. For \( \epsilon = 10^{-4} \) and L=2 m, implement this equation by paying attention to the points below:

- Read the section about the selection of \( r \) for a stiff string in your book. In the comments section of the file string_fixed_stiff.m explain briefly IN YOUR OWN WORDS why this choice is relevant. Use this \( r \) in your program.
- In the new string_fixed_stiff.m routine, let \( y(1, i)=y(2, i)=y(N-1, i)=y(N, i) \) to be equal to zero. You will start the simulation from the third point and end at point \( N-2 \). This is necessary because in order to evaluate the fourth derivative in space we need two steps behind and two steps ahead of the present step.

Send us both your propagate_stiff.m and string_fixed_stiff.m files.