Question I (20 points) : Magnetic domain walls
Solve Problem 4.8 in Bowley & Sanchez.

Question II (20 points) : One-dimensional Bose gas
Download the paper whose link I have provided below the link for the homework assignment. A Bose gas is a collection of particles (bosons) that do not interact with each other and can simultaneously occupy the same energy level regardless of its occupancy (unlike fermions that can only occupy an energy level one fermion at a time.) Starting with Eq.(14) of this paper, fill in the gaps between the equations and arrive at Eq.(24) that derives the heat capacity of the Bose gas.

Question III (20 points) : Microcanonical, canonical and grand canonical ensembles
This is a simple exercise that allows us to see once more the difference in distribution between different ensembles. As always, start with the entropy

\[ S = \sum_i p_i \ln p_i \]

(a) Assuming that the total number of microstates accessible to a given system is \( \Omega \), show that the entropy is maximum if all the states are equally likely to occur.

(b) If, on the other hand, we have an ensemble of systems sharing energy (with mean value \( \bar{E} \)), then show that the entropy is maximum when \( p_i \propto \exp(-\beta E_i) \) (\( \beta \) is a constant to be determined by the value of \( \bar{E} \)).

(c) Finally, if we have a number of systems sharing energy as well as particles (with mean values \( \bar{E} \) and \( \bar{N} \) respectively), then show that the entropy is a maximum when \( p_{ij} \propto \exp(-\alpha N_r - \beta E_s) \) (\( \alpha \) and \( \beta \) are constants determined by \( \bar{N} \) and \( \bar{E} \)).

Question IV (20 points) : Relativistic gas
Consider a gas of extremely fast particles whose energy values can be approximately by \( \varepsilon = pc \).

(a) Find the partition function.

(b) Show that the state functions is

\[ PV = \frac{1}{3} U \]

where \( U/N = 3kT \).

Question V (20 points): Anharmonic oscillator
The energy levels of a quantum mechanical, one-dimensional anharmonic oscillator may be approximated as

\[ \varepsilon_n \left( n + \frac{1}{2} \right) \hbar \omega - x \left( n + \frac{1}{2} \right)^2 \hbar \omega. \]

Find the heat capacity of \( N \) such oscillators to first order in \( x \) and fourth order in \( u \equiv \hbar \omega/kT \). Plot your result.